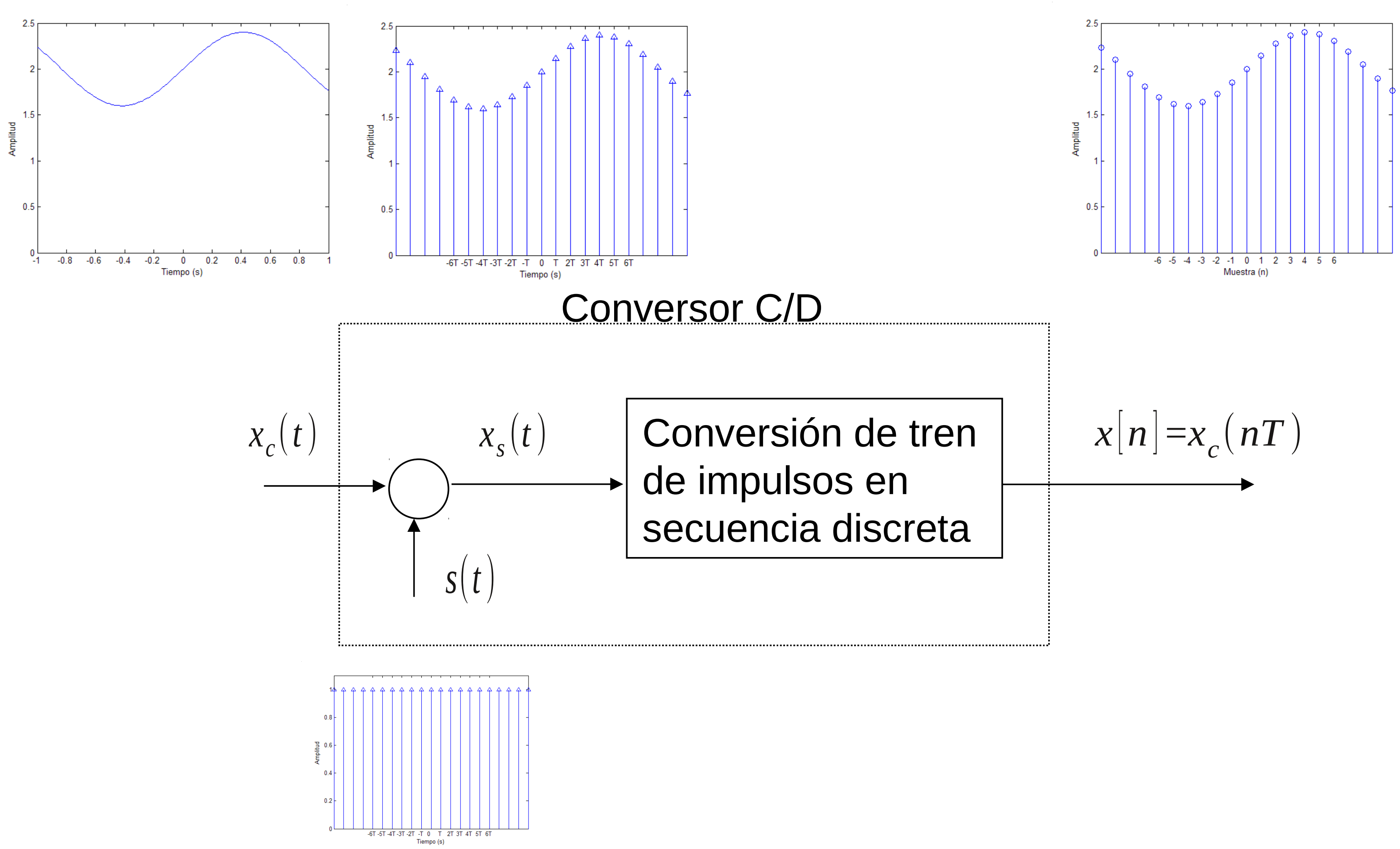
Tema 1 – Muestreo de señales continuas

3º Ingeniería Sistemas de Telecomunicación

EPS – Univ. San Pablo – CEU

Muestreo Periódico

Muestreo periódico



Representación del muestreo en el dominio de la frecuencia

Dominio del tiempo

$x_c(t)$

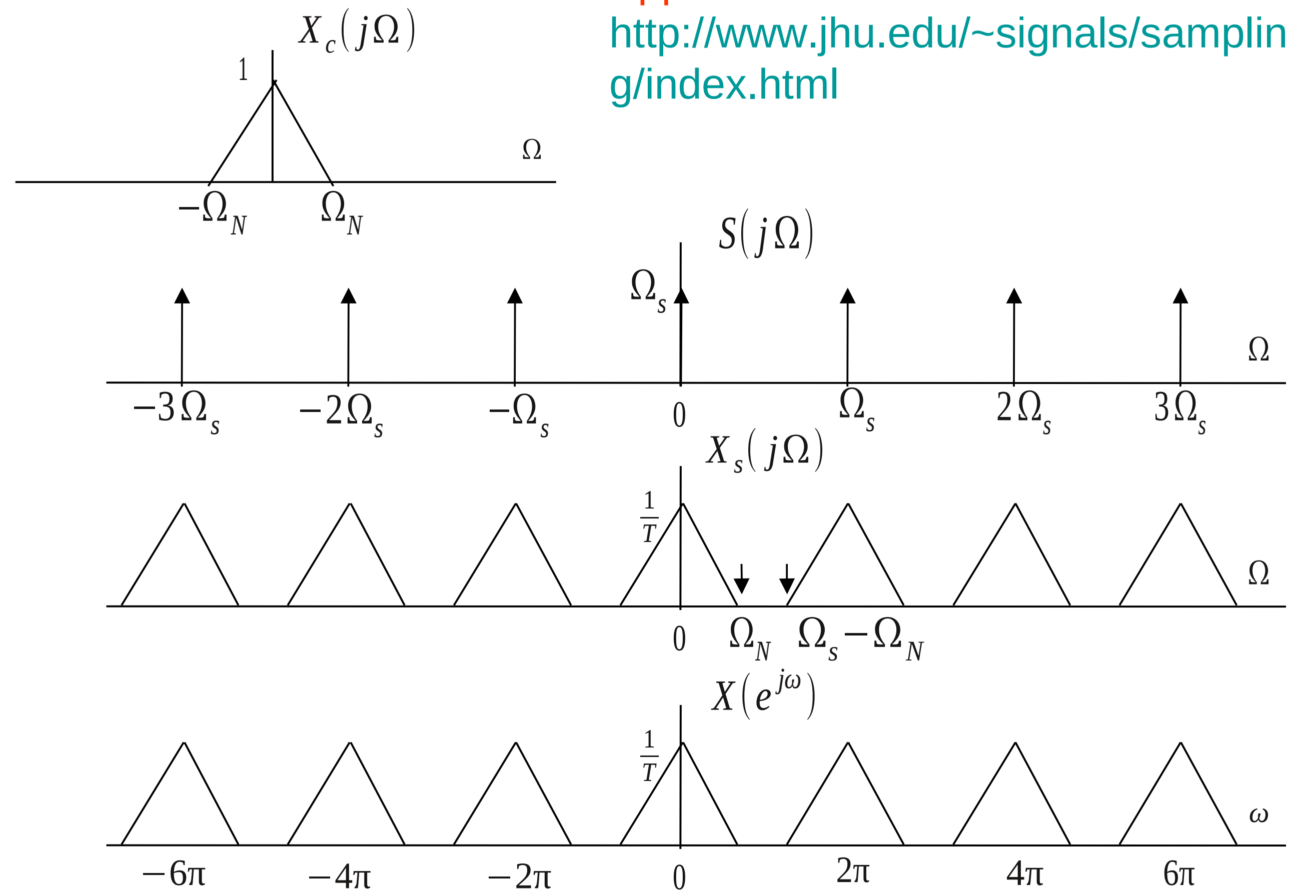
$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$

$x_s(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$

$x[n]=x_c(nT)$

Dominio de la frecuencia

Applet: http://www.jhu.edu/~signals/sampling/index.html



Representación del muestreo en el dominio de la frecuencia

Dominio del tiempo | Dominio de la frecuencia

$x_c(t)$ | $X_c(j\Omega)$

$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$ | $S(j\Omega)=\Omega_s\sum_{k=-\infty}^{\infty}\delta(\Omega-k\Omega_s)$ $\qquad \Omega_s=\frac{2\pi}{T}$ (1.1)

$x_s(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$ | $X_s(j\Omega)=\frac{1}{2\pi}X_c(j\Omega)*S(j\Omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j(\Omega-k\Omega_s))=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\Omega nT}$ (1.2)

$x[n]=x_c(nT)$ | $X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\omega n}$ (1.3)

(1.4) $X(e^{j\Omega T})=X_s(j\Omega)$

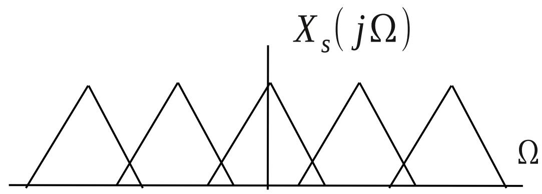
(1.5) $X(e^{j\omega})=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j(\frac{\omega}{T}-k\frac{2\pi}{T}))$

(1.6) $X(e^{j\Omega T})=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j(\Omega-k\Omega_s))$

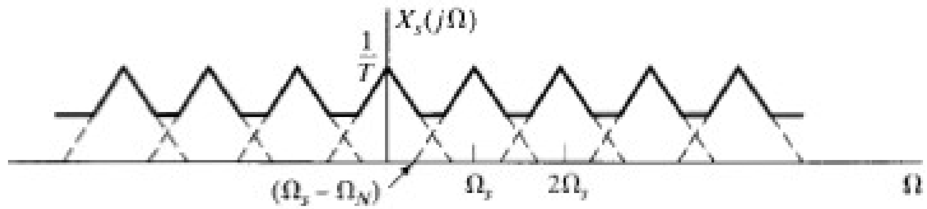
$\omega=\Omega T$

Aliasing

Se produce aliasing cuando $\Omega_N > \Omega_s - \Omega_N$ (1.7)



(c)



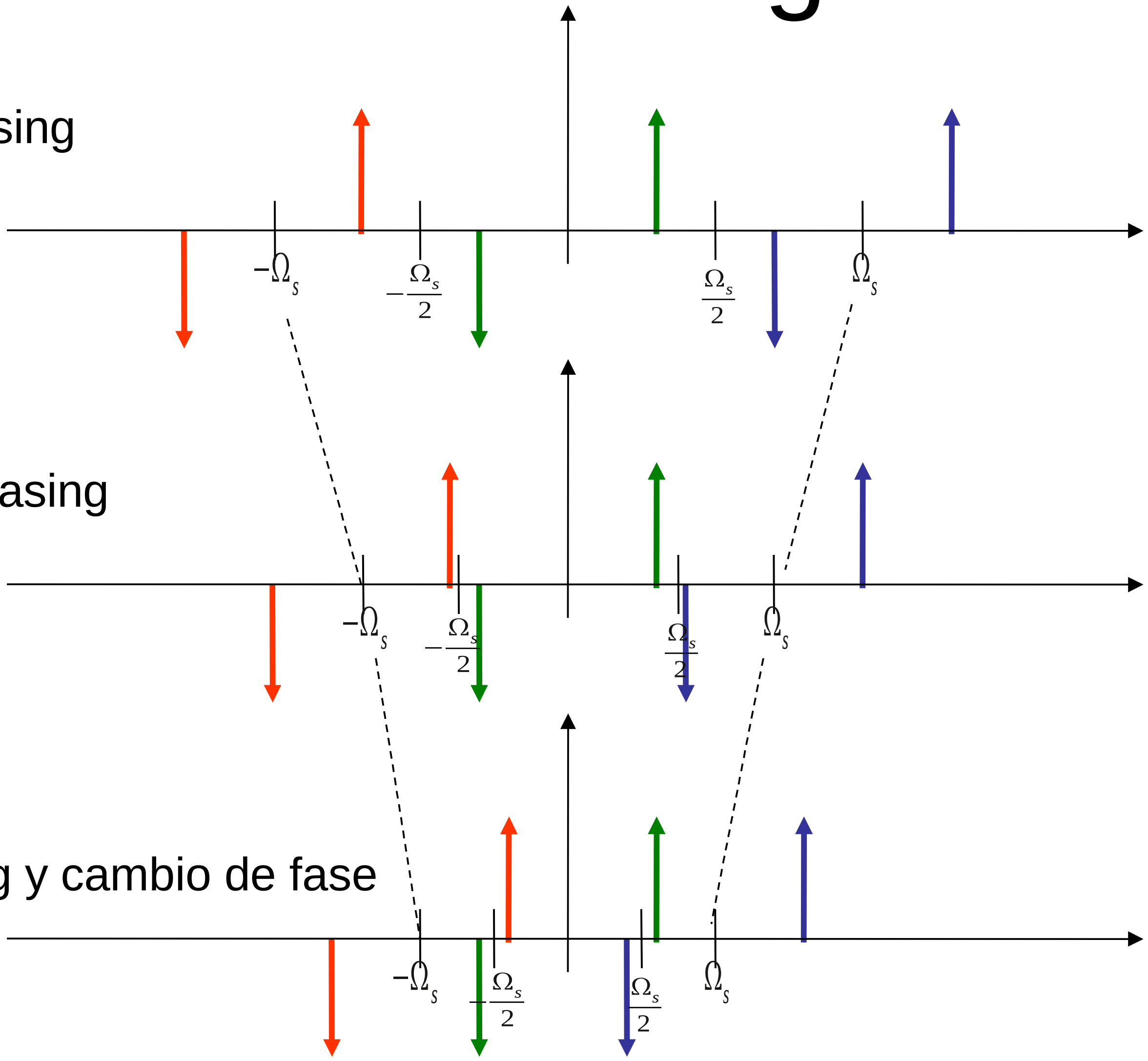
(d)

Aliasing

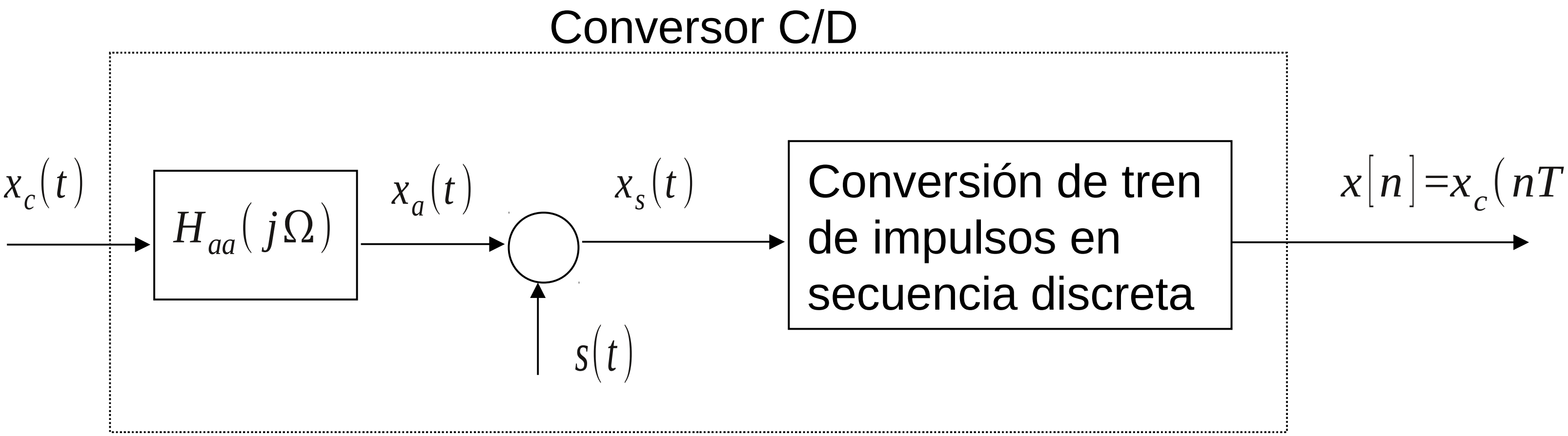
No aliasing

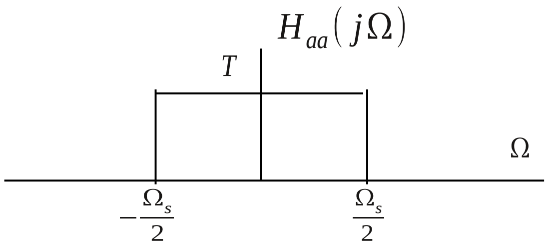
Casi aliasing

Aliasing y cambio de fase



Filtro Anti-aliasing





Reconstrucción de la señal

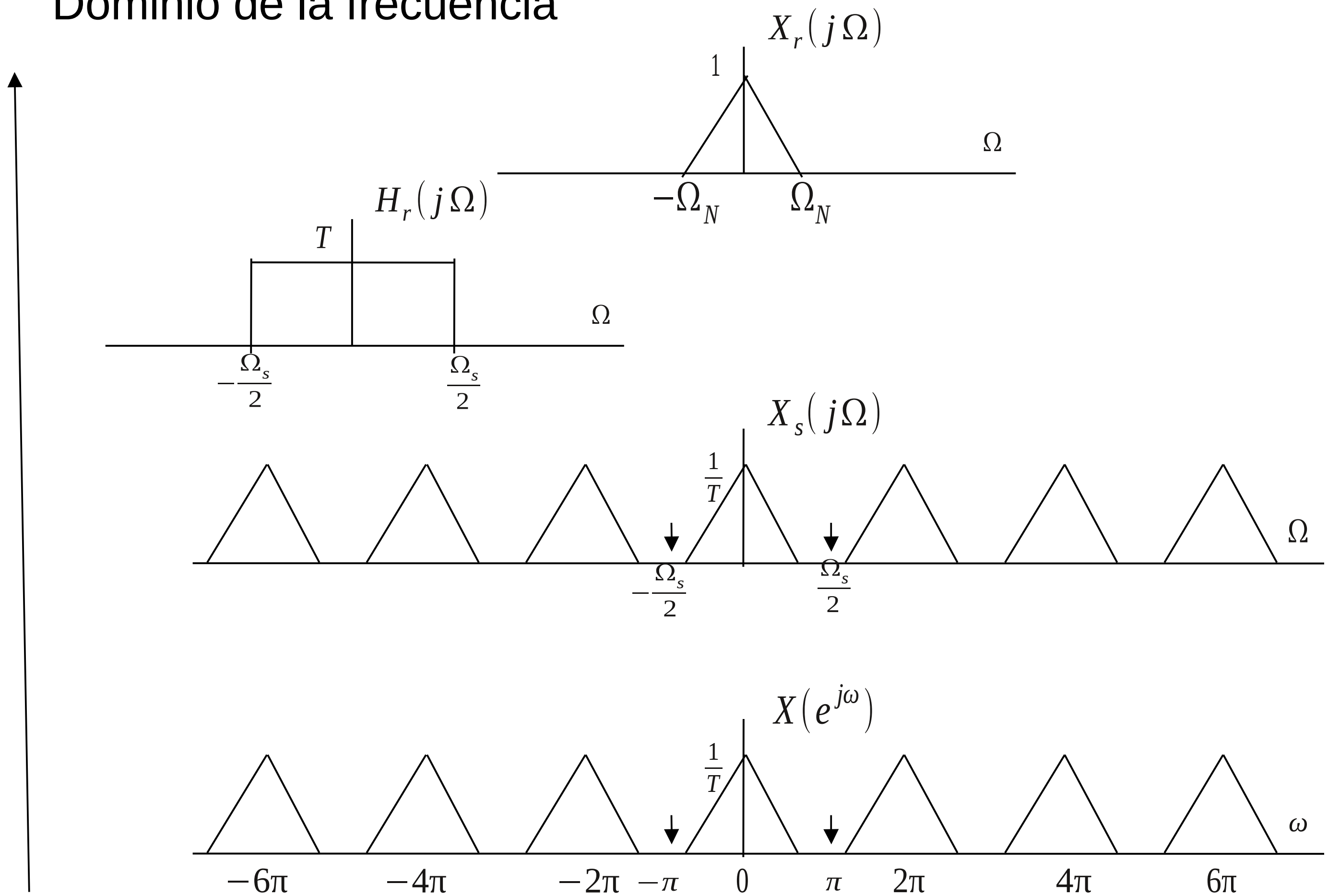
Dominio del tiempo

Dominio de la frecuencia

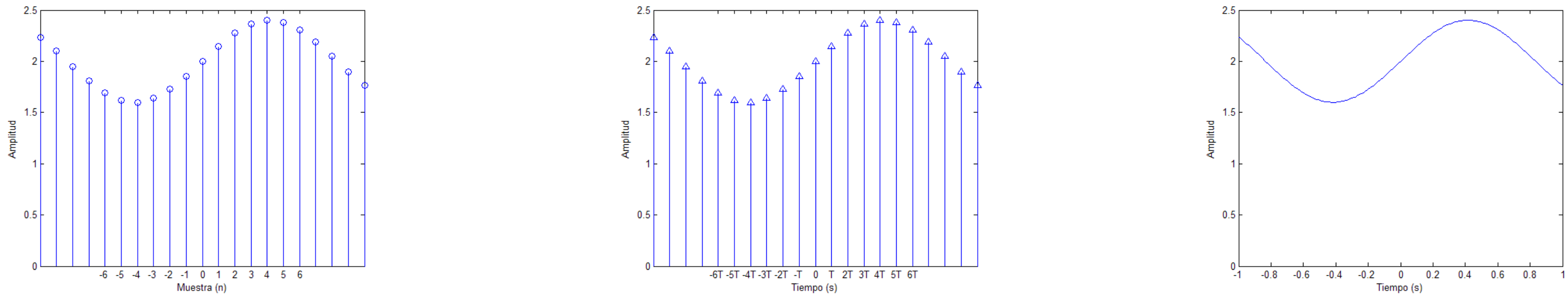
$$x_r(t) = x_s(t) * h_r(t)$$

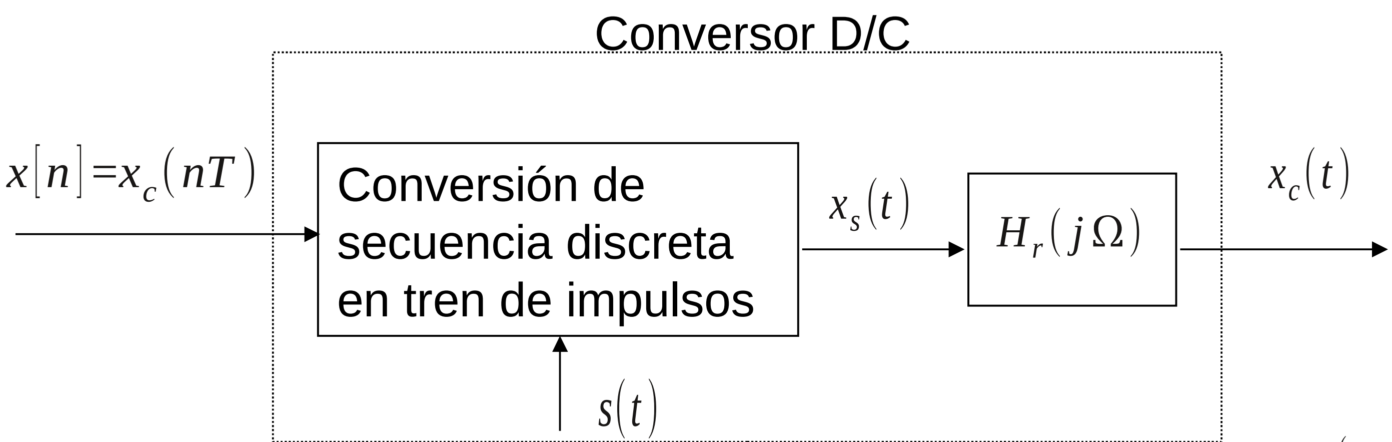
$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - nT)$$

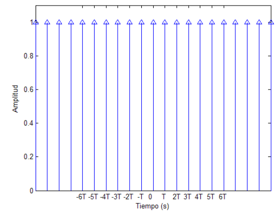
$$x[n] = x_c(nT)$$

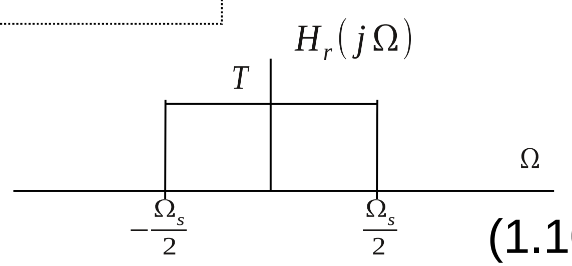


Reconstrucción de la señal









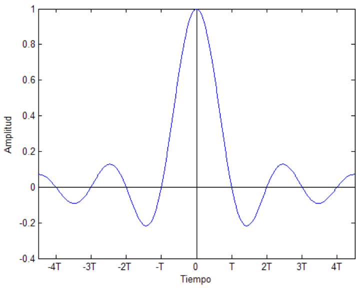
(1.10)

Reconstrucción de la señal

Dominio del tiempo

$$x_r(t) = x_s(t) * h_r(t) = \sum_{n=-\infty}^{\infty} x[n] h_r(t - nT) \quad (1.11)$$

$$h_r(t) = \operatorname{sinc}\left(\frac{t}{T}\right)$$



$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT) \quad (1.13)$$

$$x[n] = x_c(nT) \quad (1.15)$$

Dominio de la frecuencia

$$X_r(j\Omega) = H_r(j\Omega) X_s(j\Omega) = H_r(j\Omega) X\left(e^{j\Omega T}\right) \quad (1.12)$$

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n T} = X\left(e^{j\Omega T}\right) \quad (1.14)$$

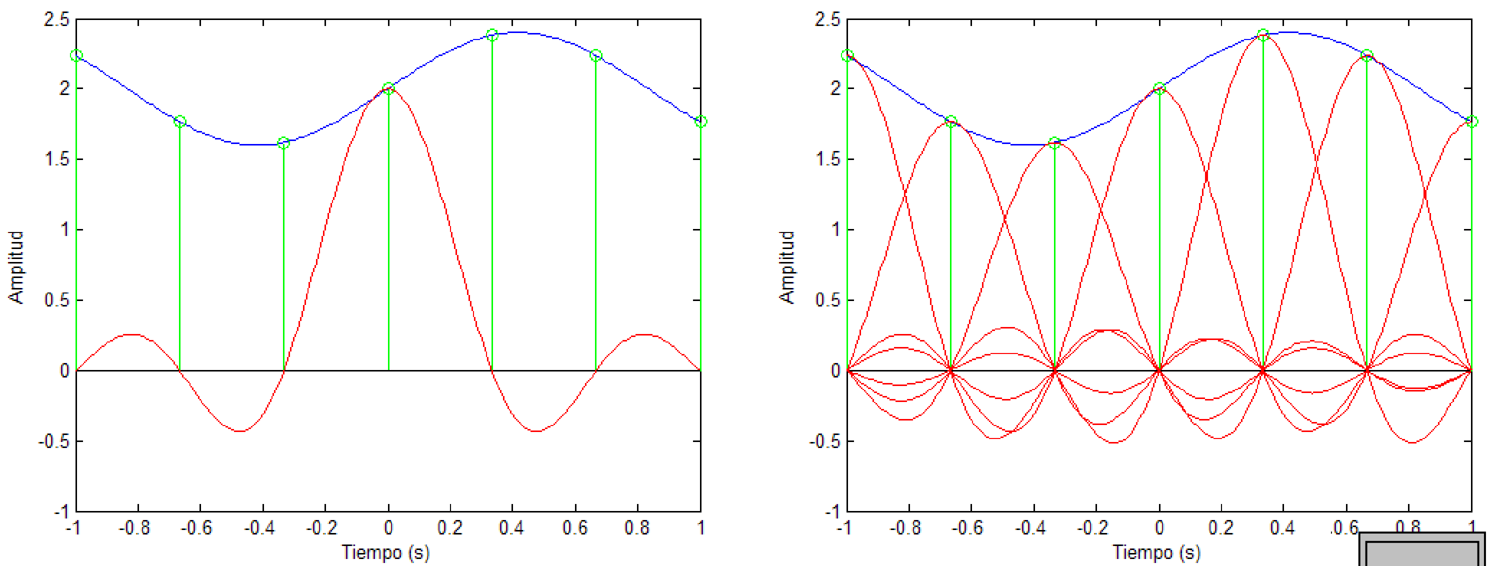
[1.3]

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad (1.16)$$

Reconstrucción de la señal

$$x_r(t) = x_s(t) * h_r(t) = \sum_{n=-\infty}^{\infty} x[n] h_r(t-nT) = \sum_{n=-\infty}^{\infty} x[n] \frac{sen[\pi(t-nT)/T]}{\pi(t-nT)/T}$$

$$h_r(t) = \operatorname{sinc}\left(\frac{t}{T}\right)$$

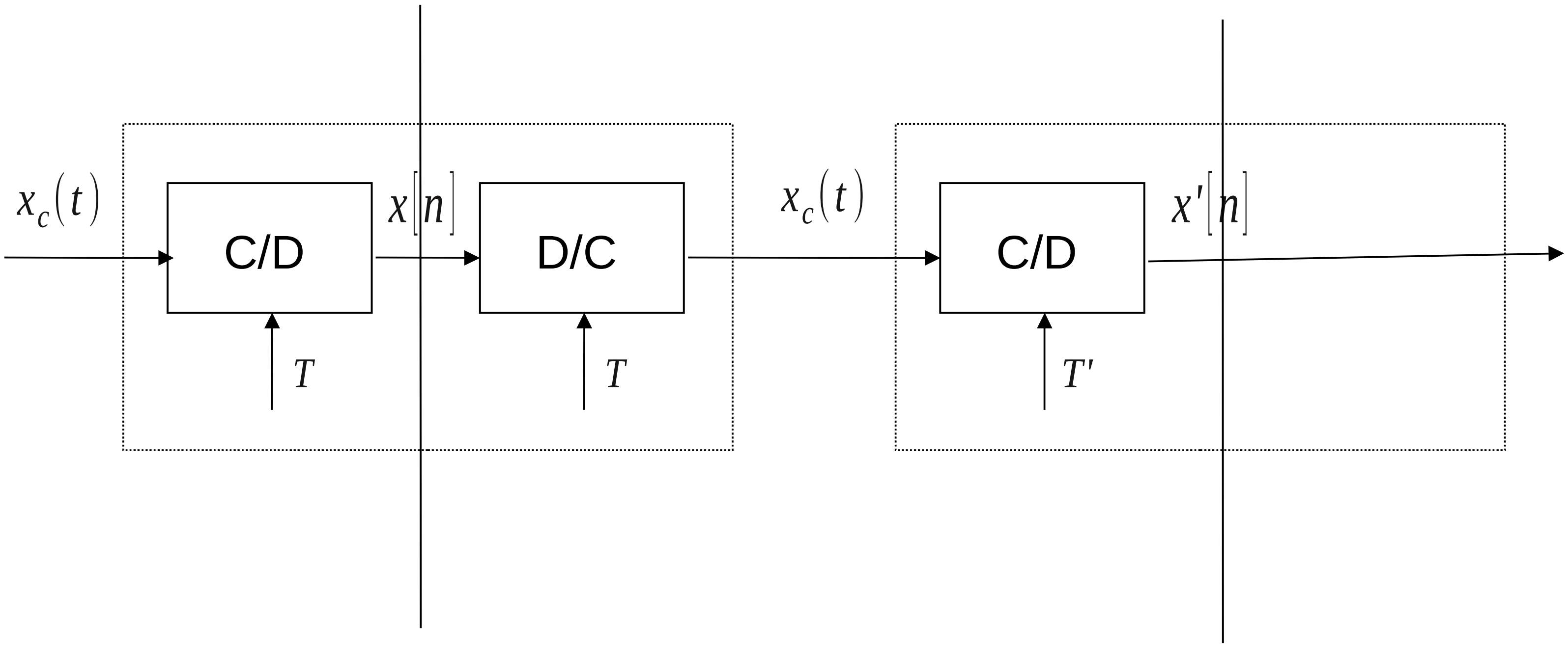


Teorema del muestreo (1.17)

Sea una señal $x_c(t)$ limitada en ancho de banda cuya frecuencia máxima es f_{max} . Entonces, esta señal se puede recuperar exactamente a partir de sus muestras tomadas a una frecuencia $\frac{1}{T}=f_s \geq 2f_{max}$ mediante la función de interpolación $h_r(t)=\sin c\left(\frac{t}{T}\right)$ .La fórmula correspondiente de interpolación es $x_r(t)=x_s(t)*h_r(t)=\sum_{n=-\infty}^{\infty} x[n]h_r(t-nT)$

Cambio de la Frecuencia de Muestreo

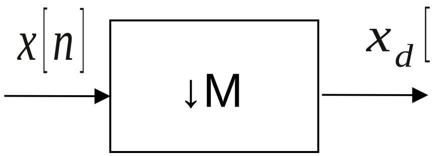
Cambio de la frecuencia de muestreo



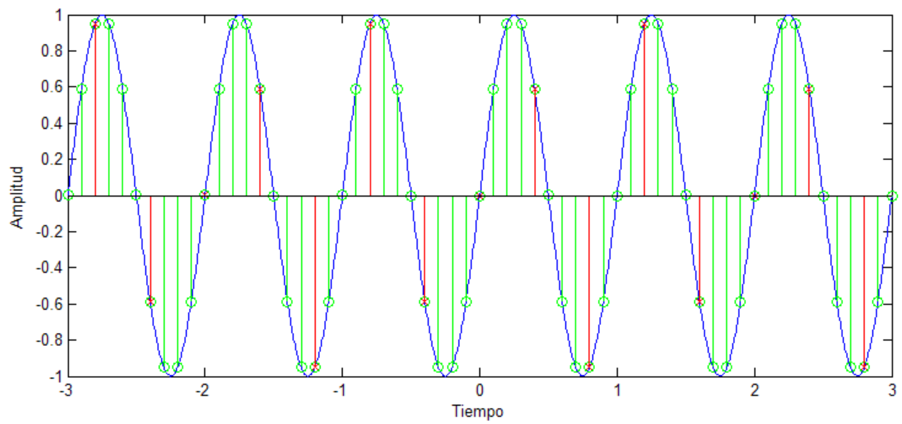
$$x[n] = x(nT)$$

$$x'[n] = x(nT')$$

Reducción de la frecuencia de muestreo



$$x_d[n] = x[nM] = x_c(nMT) \;\Rightarrow\; T' = MT \quad (1.21)$$



[1.4] $$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - k\frac{2\pi}{T}\right)\right)$$

$$r = i + kM$$

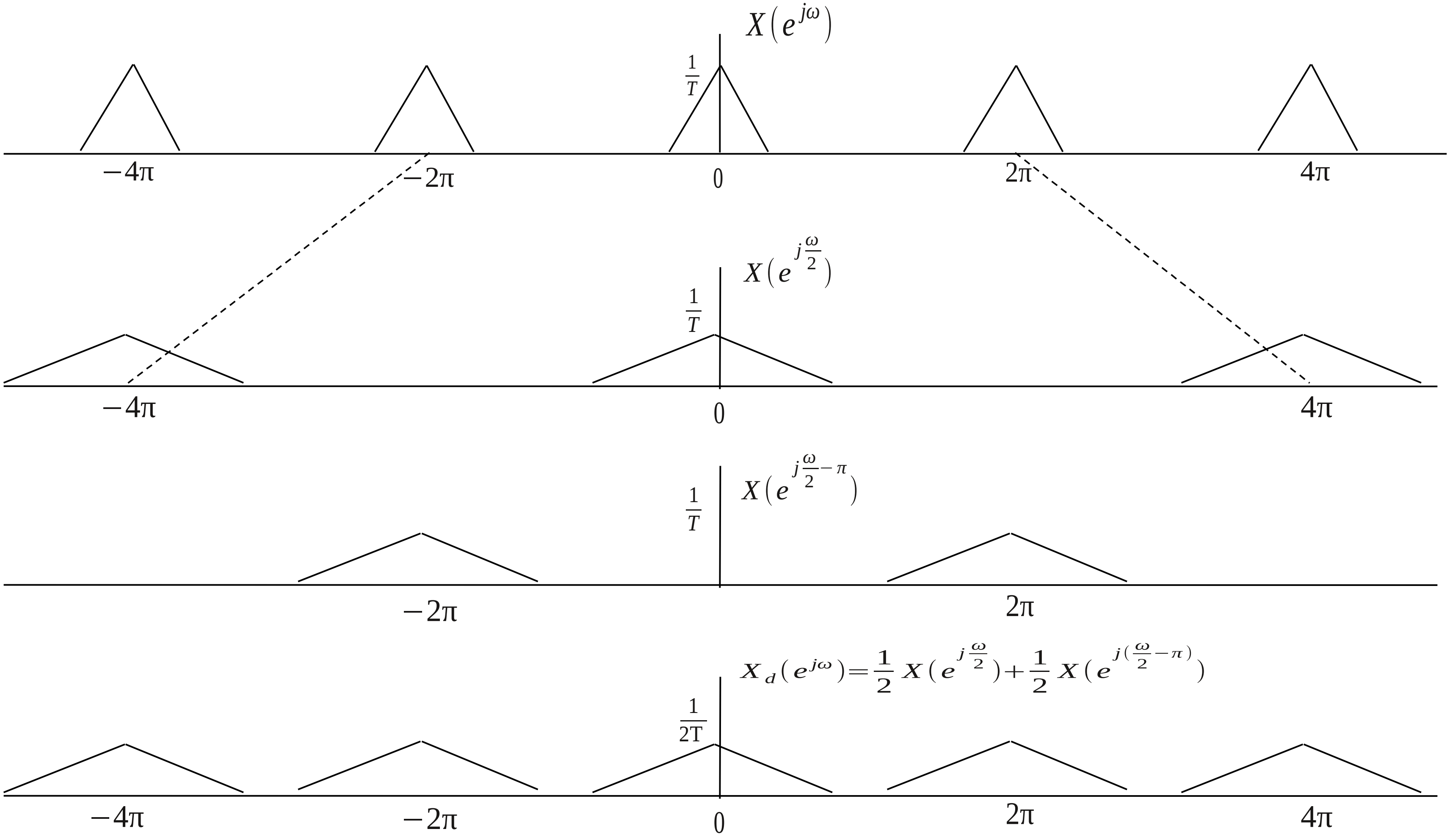
$$-\infty < k < \infty$$

$$0 \le i \le M-1$$

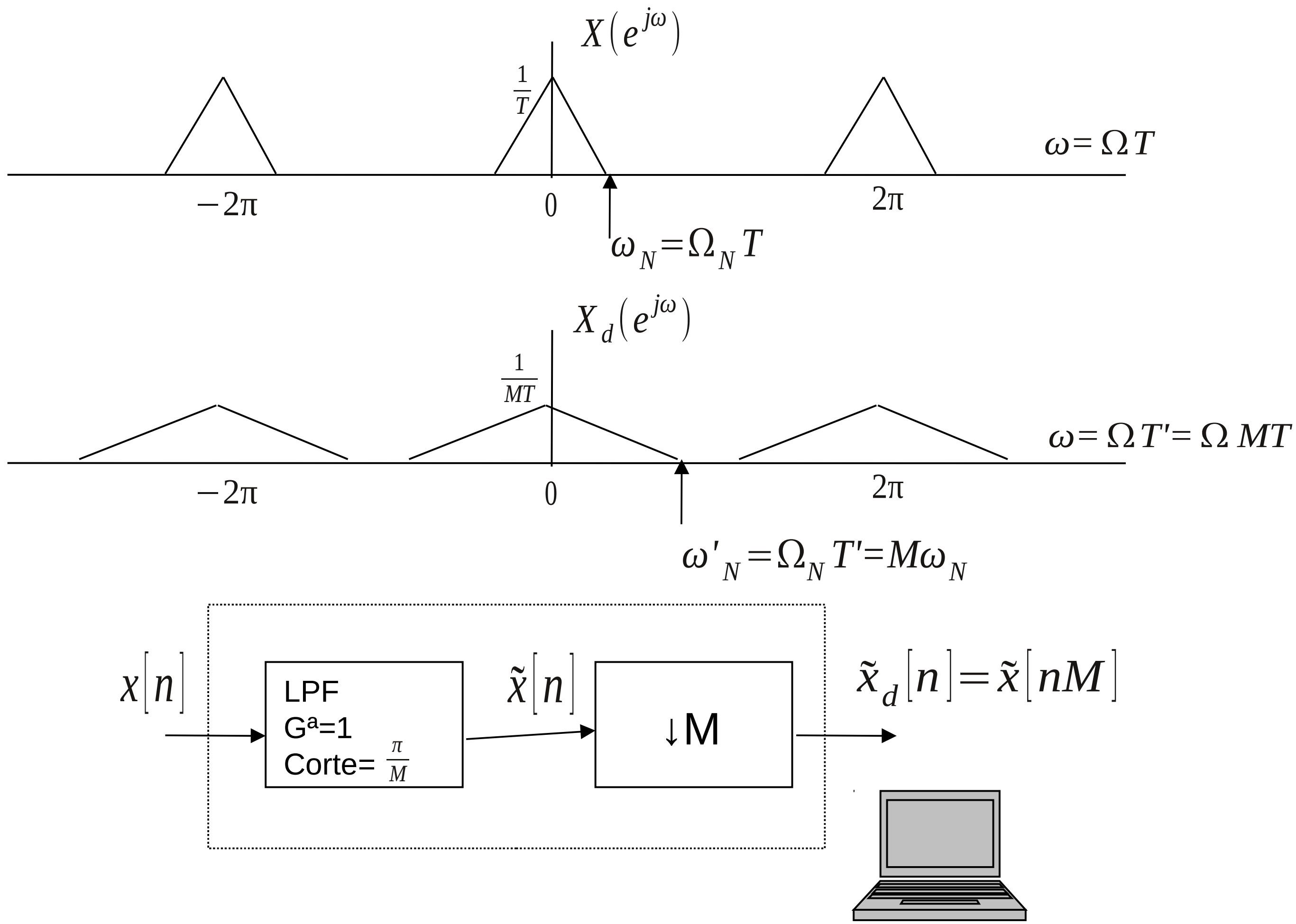
$$\boxed{X_d(e^{j\omega})} = \frac{1}{T'}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T'} - r\frac{2\pi}{T'}\right)\right) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - r\frac{2\pi}{MT}\right)\right) =$$

$$= \frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - k\frac{2\pi}{T} - i\frac{2\pi}{MT}\right)\right) = \boxed{\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)} \quad (1.22)$$

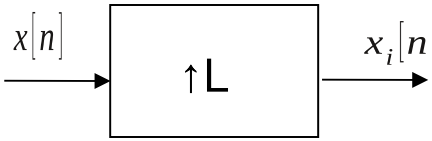
Reducción de la frecuencia de muestreo



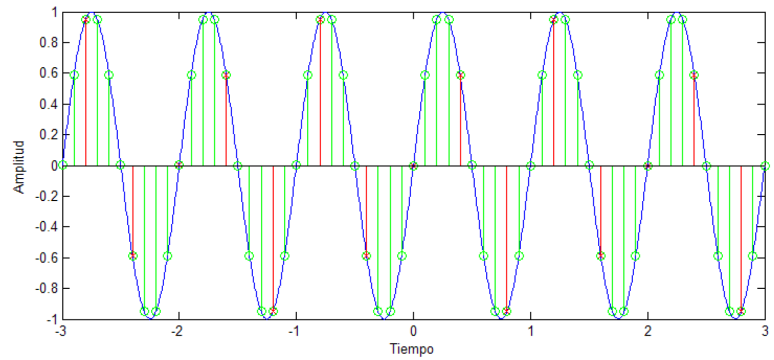
Reducción de la frecuencia de muestreo



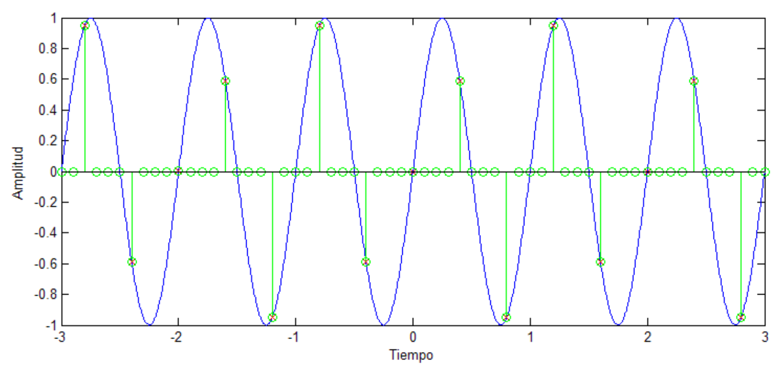
Incremento de la frecuencia de muestreo



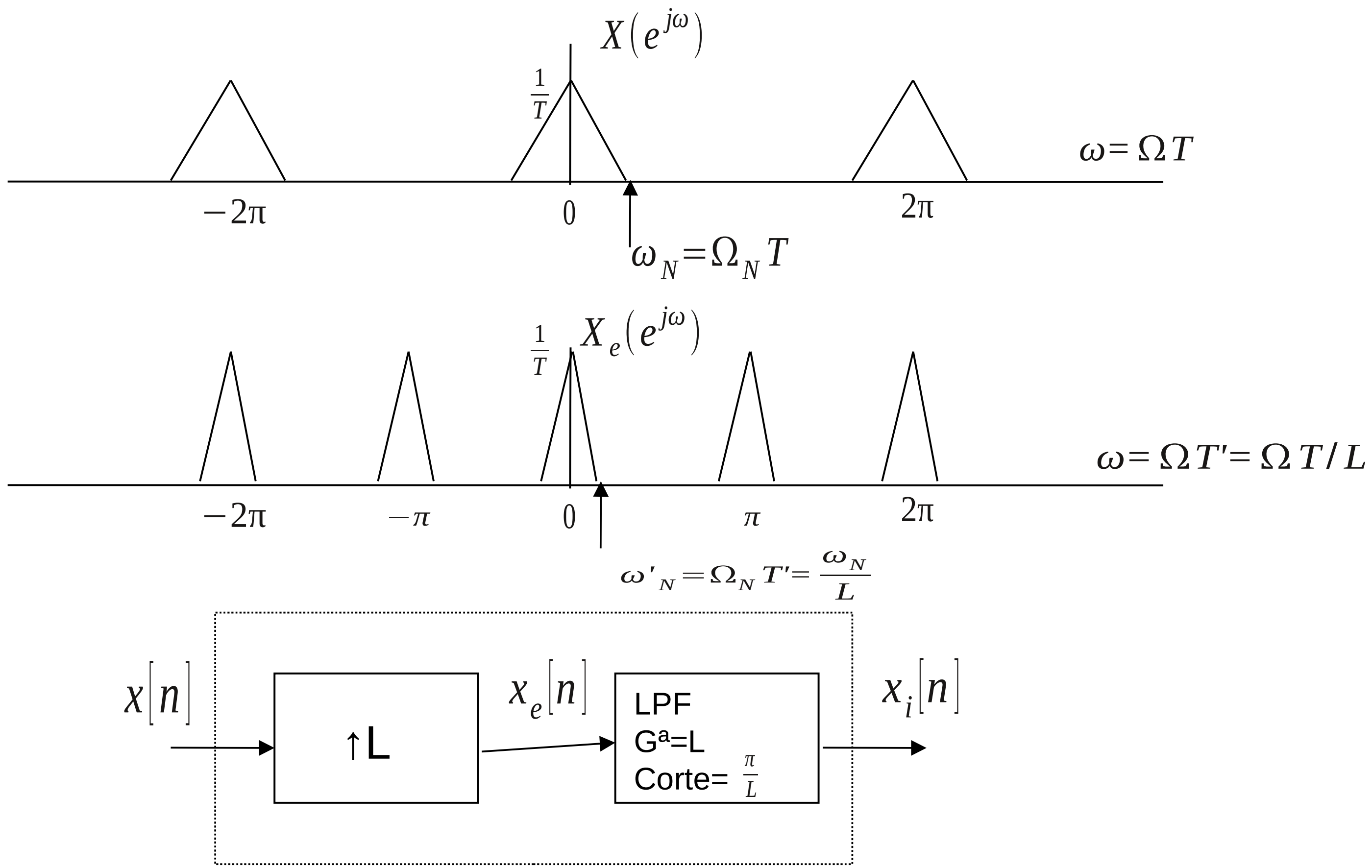
$$x_i[n] = x_c(nT/L) \Rightarrow T' = T/L \qquad (1.23)$$



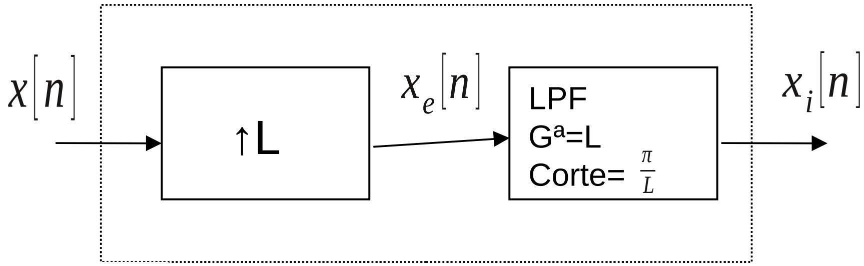
$$(1.24) \quad x_e[n] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2\mathrm{L}, \ldots \\ 0 & resto \end{cases} = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] \longleftrightarrow X_e\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} x[k]e^{j\omega kL} = X\left(e^{j\omega L}\right) \quad (1.25)$$

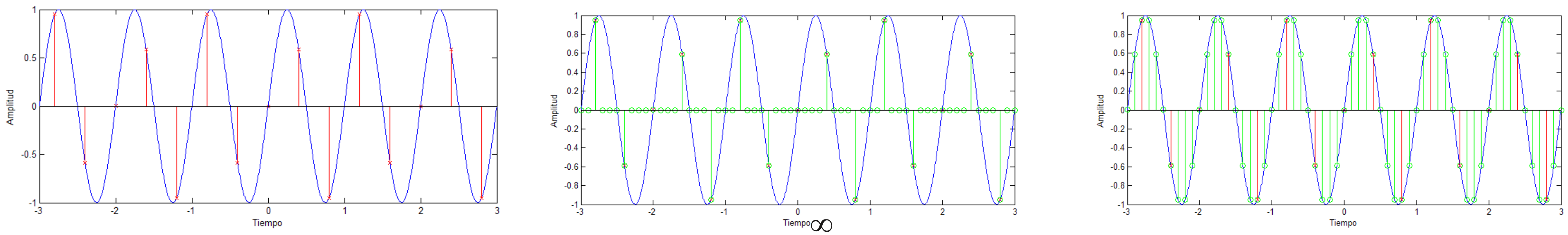


Incremento de la frecuencia de muestreo



Incremento de la frecuencia de muestreo

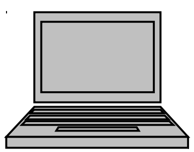


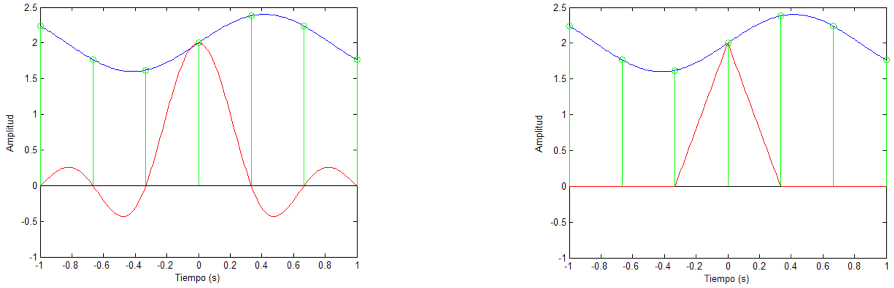


$$x_i[n]=x_e[n]*h_i[n]=\sum_{k=-\infty}^{\infty} x_e[kL]h_i[n-kL] \quad (1.26)$$

$$h_i[n]=\operatorname{sinc}\left(\frac{n}{L}\right)$$

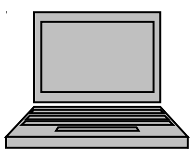
(1.27)



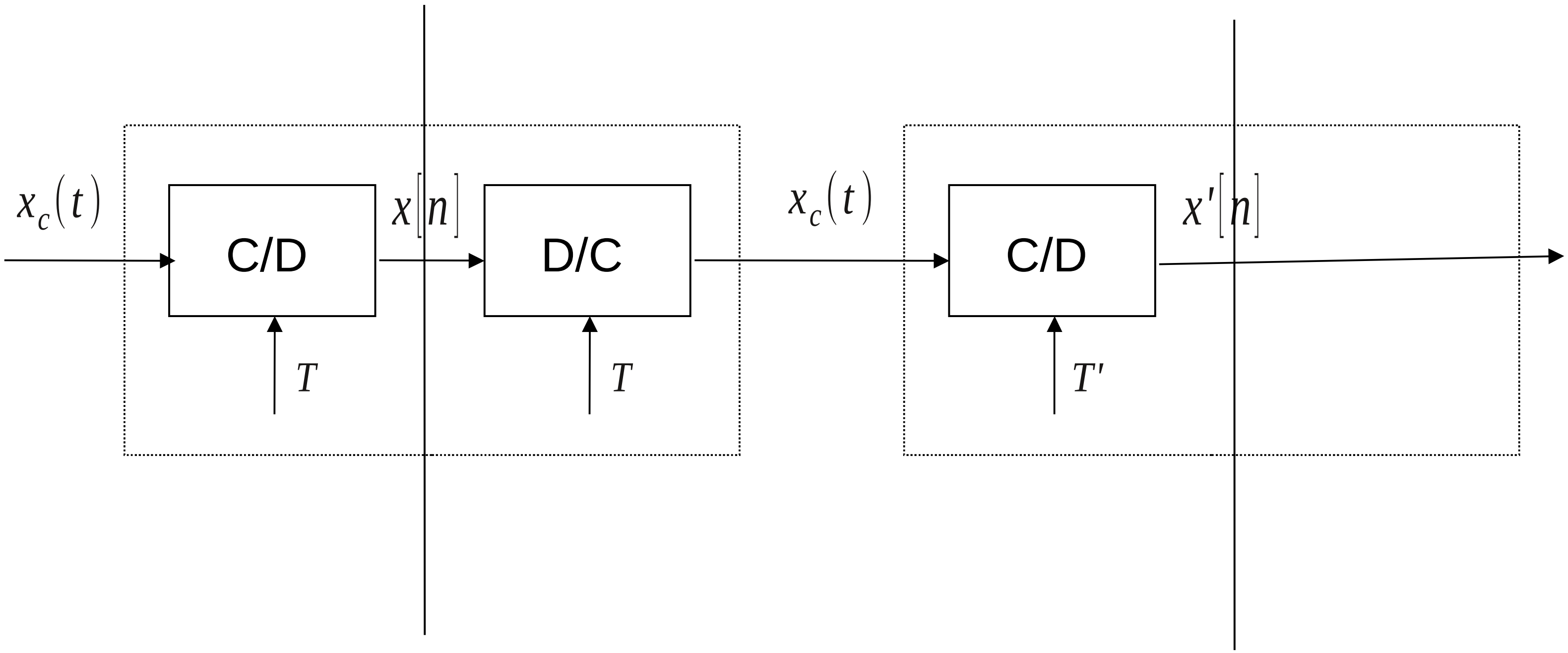


$$h_i[n]=\begin{cases}1-\frac{|n|}{L} & |n|<L\\ 0 & resto\end{cases}$$

(1.28)



Cambio de la frecuencia de muestreo por un factor racional

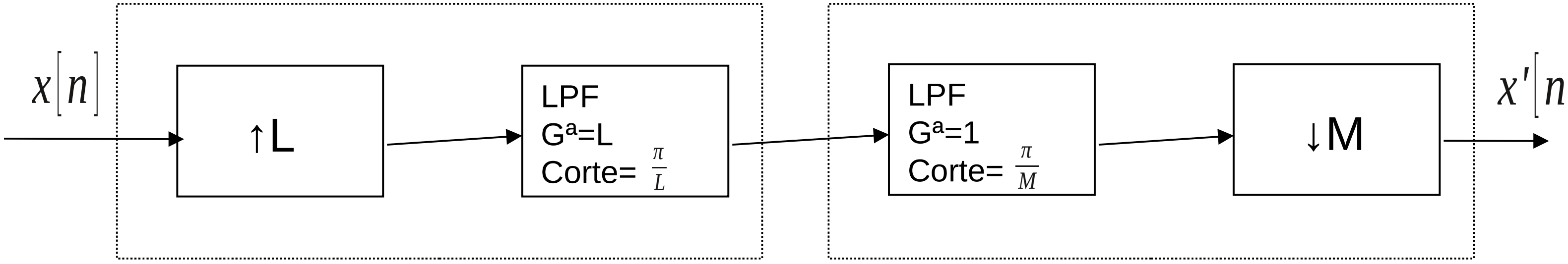


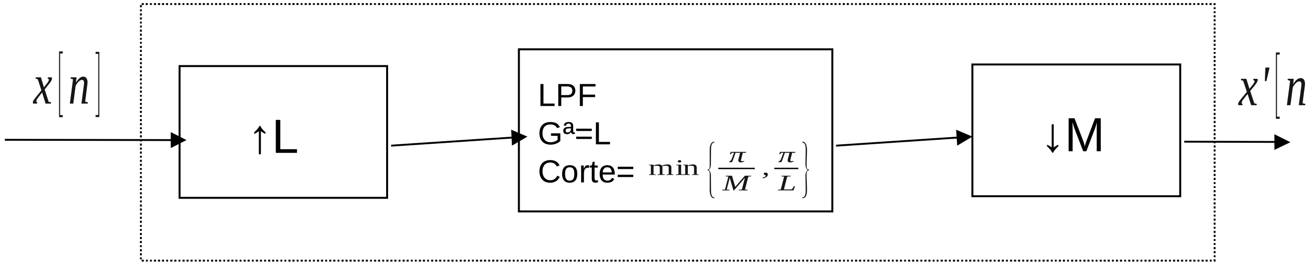
$$x[n]=x(nT)$$

$$x'[n]=x(nT')$$

$$T'=T\frac{M}{L}$$

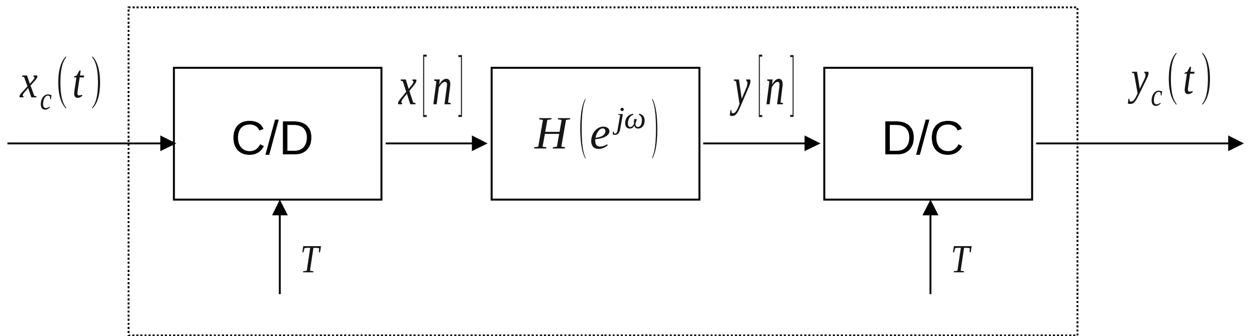
Cambio de la frecuencia de muestreo por un factor racional





Procesado discreto de señales continuas

Procesado discreto de señales continuas



[1.4]

$$X(e^{j\omega})=\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T}-k\frac{2\pi}{T}\right)\right)$$

$$Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})$$

[1.12]

$$Y_c(j\Omega)=H_r(j\Omega)Y\left(e^{j\Omega T}\right)$$

$$Y_c(j\Omega)=H_{eff}(j\Omega)X_c(j\Omega)$$

$$H_{eff}(j\Omega)=\begin{cases} H\left(e^{j\Omega T}\right) & |\Omega T|<\pi \\ 0 & resto \end{cases} \quad (1.18)$$

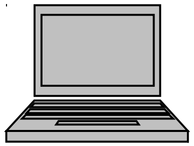
$$H(e^{j\omega})=H_{eff}\left(j\frac{\omega}{T}\right) \quad \forall\omega : |\omega|<\pi \quad (1.19)$$

Invarianza de la respuesta al impulso

$$h[n]=Th_{eff}(nT) \quad (1.20)$$

[1.4]

Procesado discreto de señales continuas

Ejemplo: 

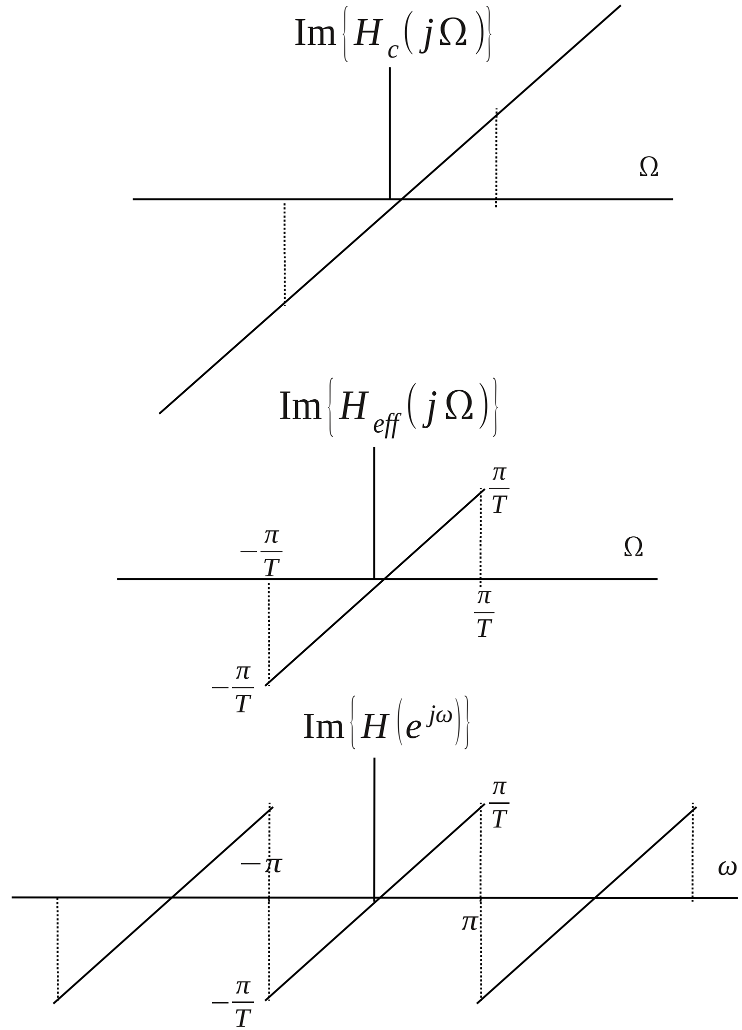
$$y_c(t)=\frac{dx_c(t)}{dt}$$

$$H_c(j\Omega)=j\Omega$$

$$H_{eff}(j\Omega)=\begin{cases} j\Omega & |\Omega T|<\pi \\ 0 & resto \end{cases}$$

$$H(e^{j\omega})=j\frac{\omega}{T} \quad \forall\,\omega:|\omega|<\pi$$

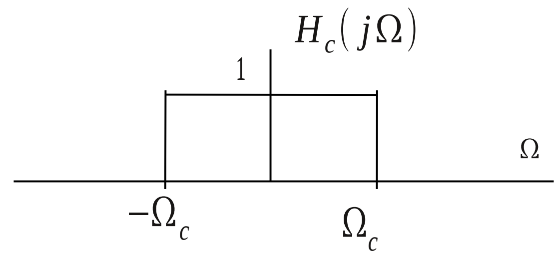
[0.125] $$h[n]=\begin{cases} 0 & n=0 \\ \dfrac{\cos \pi n}{nT} & n\neq 0 \end{cases}$$



Procesado discreto de señales continuas

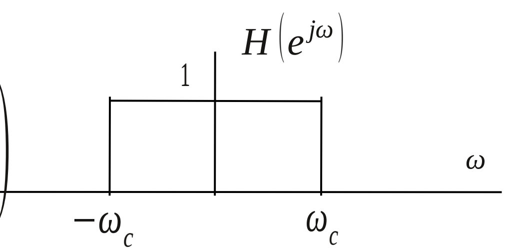
Ejemplo: 

$$H_c(j\Omega)=H_{eff}(j\Omega)=\begin{Bmatrix} 1 & |\Omega|<\Omega_c:\Omega_c T<\pi \\ 0 & resto \end{Bmatrix}$$



$$h(t)=\frac{\Omega_c}{\pi}\sin c\left(\frac{\Omega_c}{\pi}t\right)$$

$$h[n]=Th(nT)=T\frac{\Omega_c}{\pi}\sin c\left(\frac{\Omega_c}{\pi}nT\right)=\frac{\omega_c}{\pi}\sin c\left(\frac{\omega_c}{\pi}n\right)$$



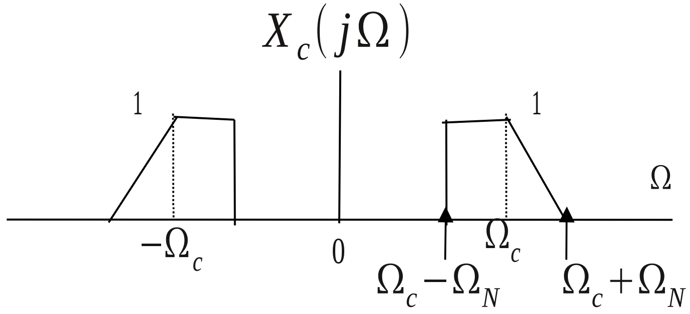
$$H(e^{j\omega})=\begin{Bmatrix} 1 & |\omega|<\omega_c:\omega_c<\pi \\ 0 & resto \end{Bmatrix}$$

$$\omega_c=\Omega_c T$$
$$-\pi\leq\omega<\pi$$

[1.11]

Muestreo de señales paso banda

Muestreo de señales paso banda



$\Omega_s \geq 2(\Omega_c+\Omega_N)$!!

Solución: Muestrear el equivalente paso bajo

[0.153] $x(t)=x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t$

El ancho de banda de la señal equivalente paso bajo es Ω_N .
La idea es muestrear la componente en fase y en cuadratura por separado

Supongamos que $\Omega_c+\Omega_N=k(2\Omega_N)$ (1.52)

$\Omega_s=2(2\Omega_N)$ (1.53)

Nyquist

Muestreo de señales paso banda

[0.153] $x(t) = x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t$

(1.54) $x[n] = x(nT) = x_c(nT)\cos\Omega_c nT - x_s(nT)\sin\Omega_c nT =$

$$\left\{\begin{array}{l}\Omega_c + \Omega_N = k(2\Omega_N) \\ \Omega_s = 2(2\Omega_N) = \frac{2\pi}{T}\end{array}\right.$$

[1.52,1.53]

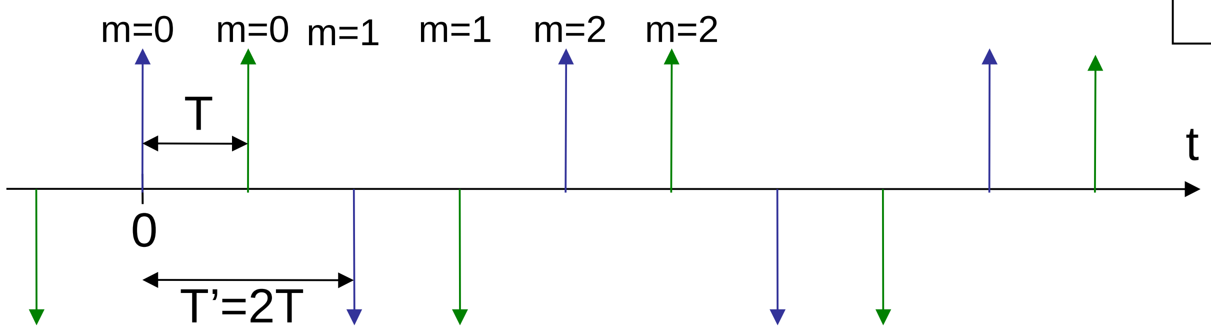
$$= x_c(nT)\cos\left(\pi n\frac{2k-1}{2}\right) - x_s(nT)\sin\left(\pi n\frac{2k-1}{2}\right) =$$

$$= \begin{cases} x_c(mT')(-1)^m & n = 2m \\ (-1)^{k+1} x_s(mT'-T)(-1)^m & n = 2m-1 \end{cases}$$

$\Omega_c + \Omega_N = k(2\Omega_N) \longrightarrow \Omega_N = \frac{\Omega_c}{2k-1}$

$\Omega_s = 2(2\Omega_N) = \frac{2\pi}{T} \longrightarrow \Omega_N = \frac{\pi}{2T}$

$$\frac{\Omega_c}{2k-1} = \frac{\pi}{2T} \Rightarrow \Omega_c T = \pi\frac{2k-1}{2}$$

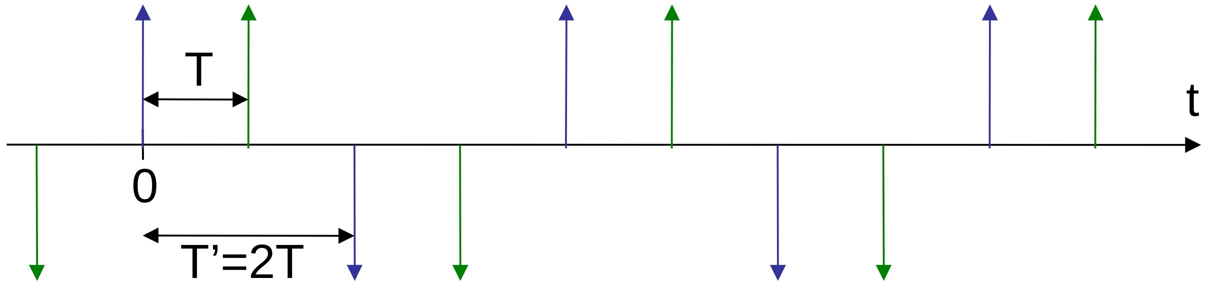


$\Omega_s' = 2\Omega_N$

(1.55)

Muestreo de señales paso banda

Reconstrucción de cada una de las componentes



$$x[0]=x_c[0](-1)^0$$
$$x[1]=x_s[0](-1)^{k+1}$$
$$x[2]=x_c[1](-1)^1$$
$$x[3]=x_s[1](-1)^{1+k+1}$$
$$x[4]=x_c[2](-1)^2$$
$$x[5]=x_s[2](-1)^{2+k+1}$$

(1.56) $x_c[m]=x[2m](-1)^m$

$x_s[m]=x[2m+1](-1)^{m+k+1}$ (1.57)

(1.58) $x_c(t)=\sum_{m=-\infty}^{\infty} x_c[m]h_r(t-mT')$

$x_s(t)=\sum_{m=-\infty}^{\infty} x_s[m]h_r(t-(mT'+T))$ (1.59)

Muestreo de señales paso banda

$$x(t)=x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t=$$ [0.153,1.58,1.59]

$$=\sum_{m=-\infty}^{\infty}\left(x_c[m]h_r(t-mT')\cos\Omega_c t - x_s[m]h_r(t-(mT'+T))\sin\Omega_c t\right)=$$

$$=\sum_{m=-\infty}^{\infty}\left(x[2\mathrm{m}](-1)^m h_r(t-mT')\cos\Omega_c t - x[2\mathrm{m}+1](-1)^{m+k+1}h_r(t-(mT'+T))\sin\Omega_c t\right)=$$

$$\cos(a+b)=\cos a\cos b+\sin a\sin b$$

$$\Omega_c T=\pi\frac{2\mathrm{k}-1}{2}$$

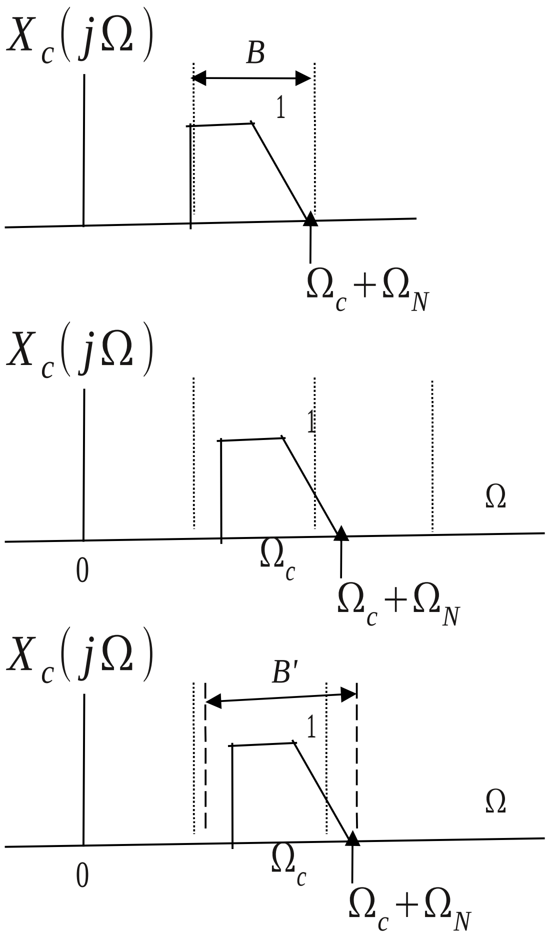
$$=\sum_{m=-\infty}^{\infty}\left(x[2\mathrm{m}]h_r(t-2mT)\cos\Omega_c(t-2mT) - x[2\mathrm{m}+1]h_r(t-(2\mathrm{m}+1)T)\cos\Omega_c(t-(2\mathrm{m}+1)T)\right)=$$

$$=\sum_{n=-\infty}^{\infty}x[n]h_r(t-nT)\cos\Omega_c(t-nT)$$

(1.60)

Suponiendo que $\Omega_c+\Omega_N=k(2\Omega_N)$ [1.52]

Muestreo de señales paso banda



Supongamos que $\Omega_c+\Omega_N=kB$

Supongamos que $\Omega_c+\Omega_N\neq kB$ (1.61)

Entonces $k=\left\lfloor \dfrac{\Omega_c+\Omega_N}{B} \right\rfloor$ (1.62)

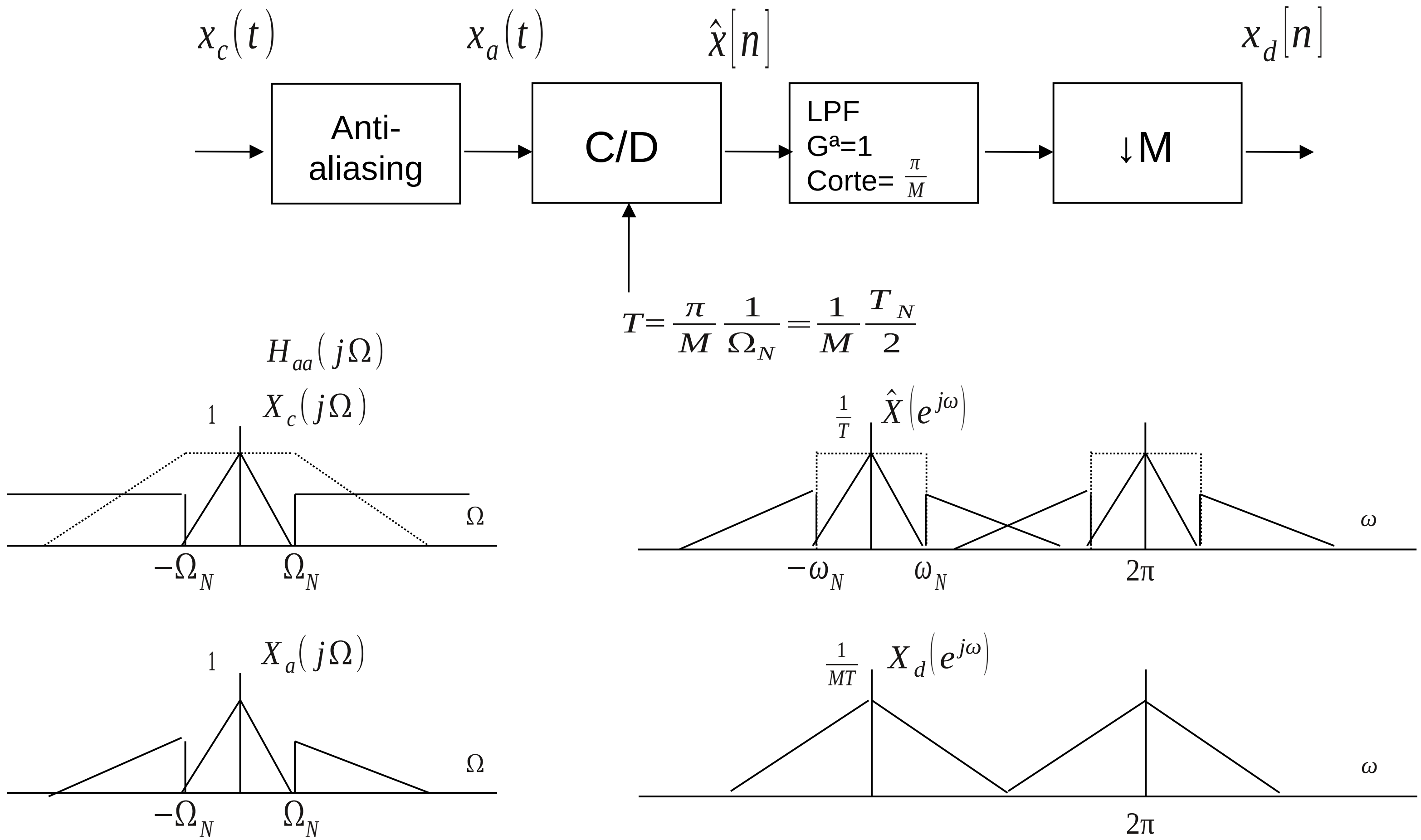
$$B'=2\,\Omega_N'=\frac{\Omega_c+\Omega_N}{k} \quad (1.63)$$

$$\Omega_c'+\Omega_N'=kB' \quad (1.64)$$

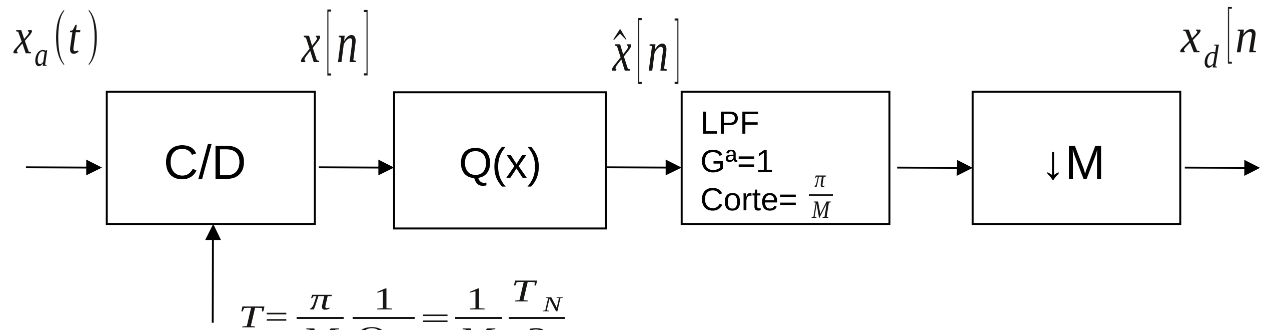
$$2(2\,\Omega_N)\leq\Omega_s'<4(2\,\Omega_N) \quad (1.65)$$

Aplicaciones del Oversampling

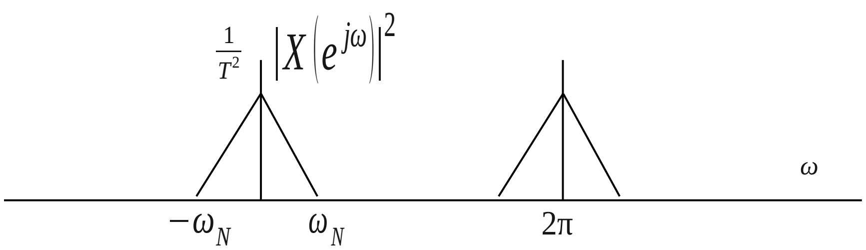
Aplicaciones del oversampling: Filtros antialiasing

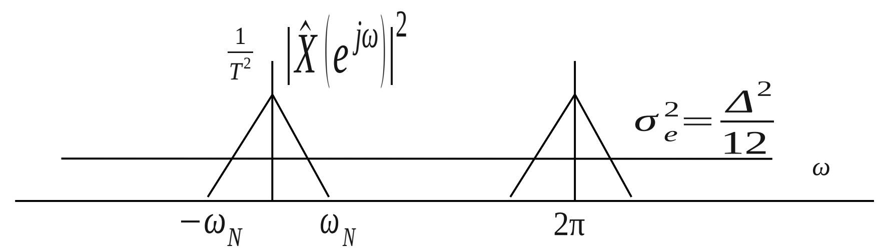


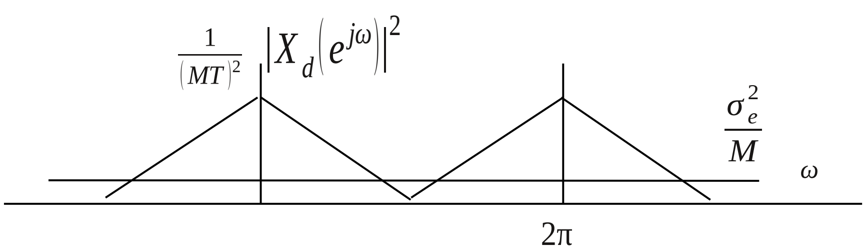
Aplicaciones del oversampling: Reducción del ruido de muestreo



$$T=\frac{\pi}{M}\frac{1}{\Omega_N}=\frac{1}{M}\frac{T_N}{2}$$

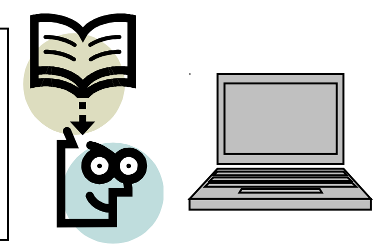






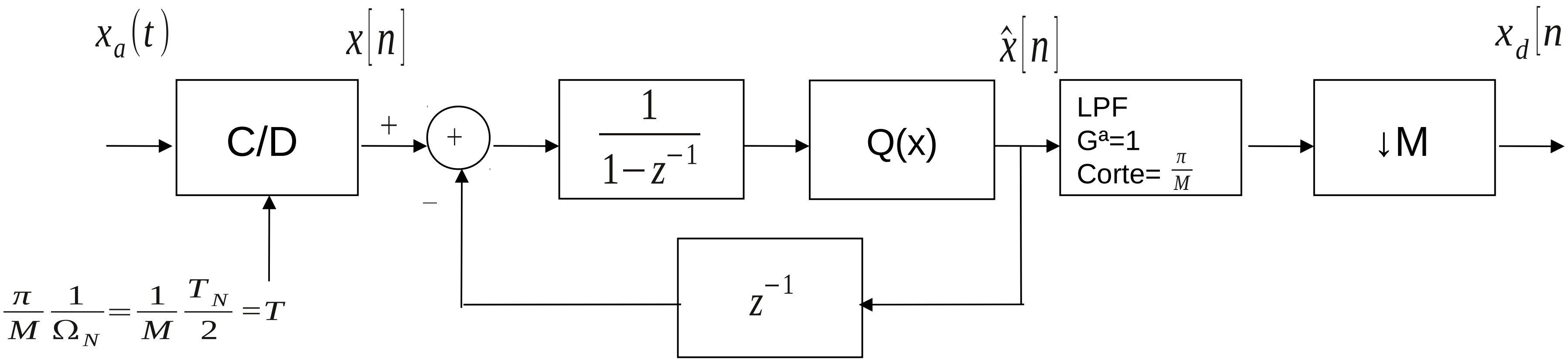
$$E\left\{x_d^2[n]\right\}=E\left\{x^2[n]\right\}$$

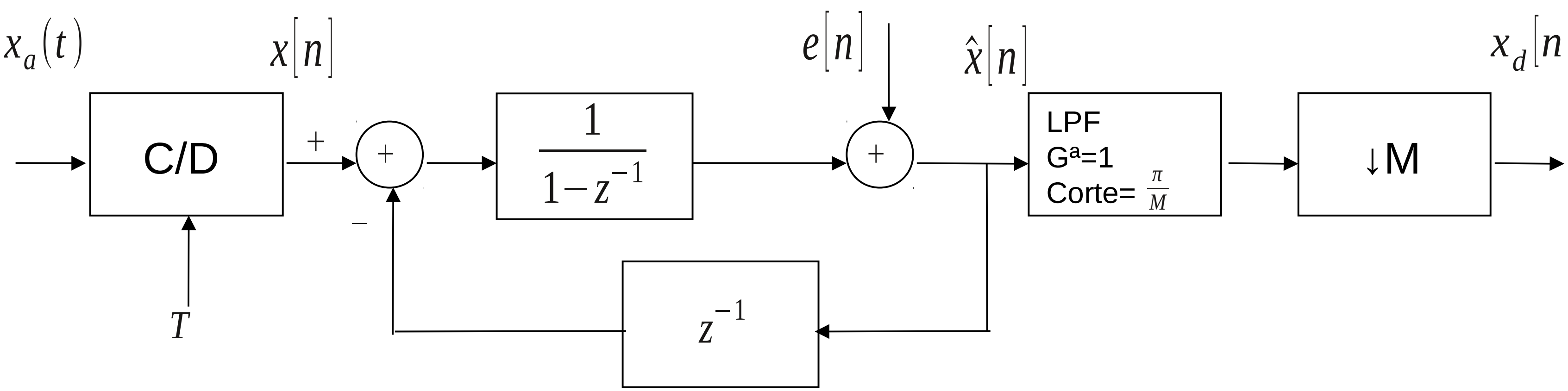
$$E\left\{e_d^2[n]\right\}=\frac{1}{M}E\left\{e^2[n]\right\}$$



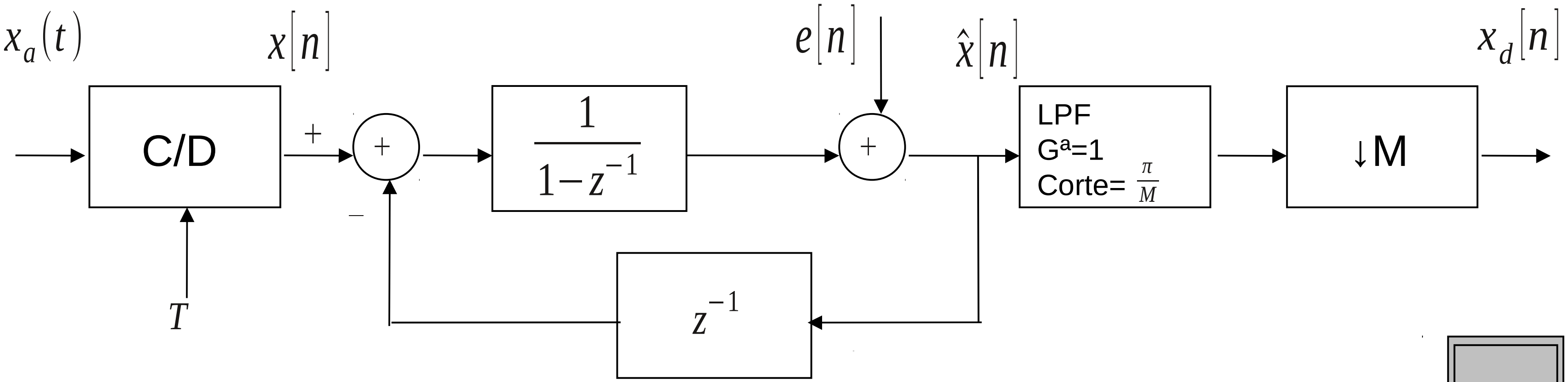
(1.66, 1.67)

Aplicaciones del oversampling: Noise shaping





Aplicaciones del oversampling: Noise shaping



$$\hat{X}(z)=(X(z)-z^{-1}\hat{X}(z))\frac{1}{1-z^{-1}}+E(z)$$

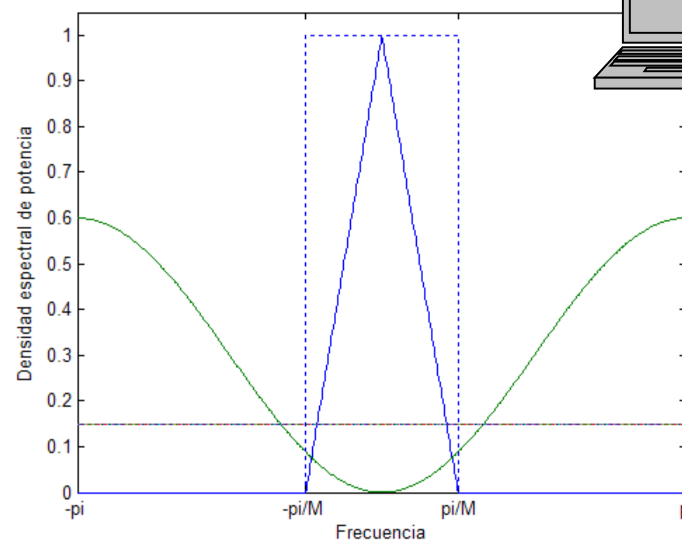
(1.68) $\hat{X}(z)=X(z)+(1-z^{-1})E(z)$

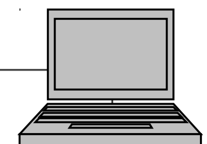
(1.69) $\hat{x}[n]=x[n]+e[n]-e[n-1]=x[n]+e'[n]$

$$\sigma_{e'}^2=2\sigma_e^2$$

(1.70)

(1.71) $S_{e'}(e^{j\omega})=|1-e^{-j\omega}|^2\sigma_e^2=\left(2\sin\frac{\omega}{2}\right)^2\sigma_e^2$





Tema 1 – Muestreo de señales continuas

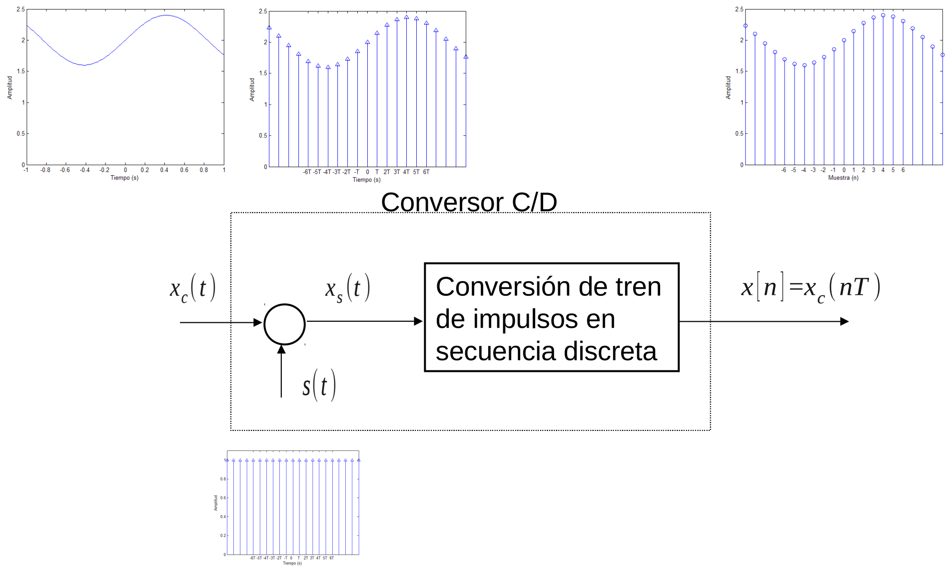
3º Ingeniería Sistemas de Telecomunicación

EPS – Univ. San Pablo – CEU

Oppenheim II (Cap. 4). Proakis (Cap. 9). Oppenheim I (Cap. 7)

Muestreo Periódico

Muestreo periódico



TDS EPS-San Pablo CEU 3

Bibliografía: Opp 4.1

Problemas Opp: 4.1

Representación del muestreo en el dominio de la frecuencia

Dominio del tiempo

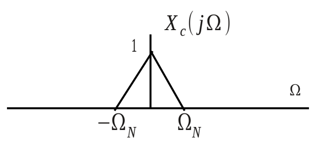
$x_c(t)$

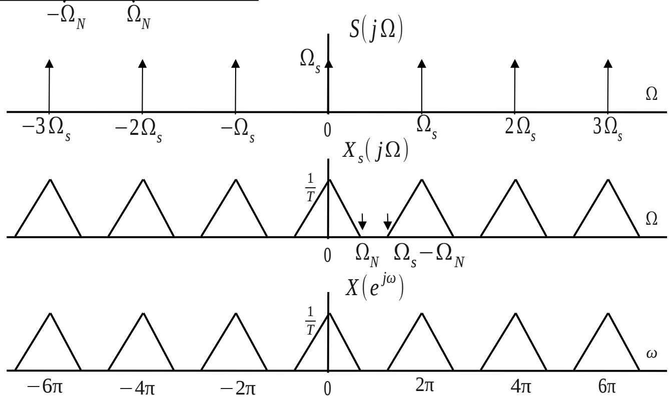
$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT)$

$x[n] = x_c(nT)$

Dominio de la frecuencia





Applet: http://www.jhu.edu/~signals/sampling/index.html

TDS EPS-San Pablo CEU 4

Bibliografía: Opp 4.2

Problemas Opp: 4.7*, 4.10, 4.11

Problemas Pro: 1.15, 9.6

Representación del muestreo en el dominio de la frecuencia

Dominio del tiempo	Dominio de la frecuencia	
$x_c(t)$	$X_c(j\Omega)$	
$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$S(j\Omega)=\Omega_s\sum_{k=-\infty}^{\infty}\delta(\Omega-k\Omega_s)$ $\quad \Omega_s=\frac{2\pi}{T}$	(1.1)
$x_s(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$	$X_s(j\Omega)=\frac{1}{2\pi}X_c(j\Omega)*S(j\Omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j(\Omega-k\Omega_s))=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\Omega nT}$	(1.2)
$x[n]=x_c(nT)$	$X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\omega n}$	(1.3)

(1.4) $$X(e^{j\Omega T})=X_s(j\Omega)$$

(1.5) $$X(e^{j\omega})=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c\left(j\left(\frac{\omega}{T}-k\frac{2\pi}{T}\right)\right)$$

(1.6) $$X(e^{j\Omega T})=\frac{1}{T}\sum_{k=-\infty}^{\infty}X_c(j(\Omega-k\Omega_s))$$

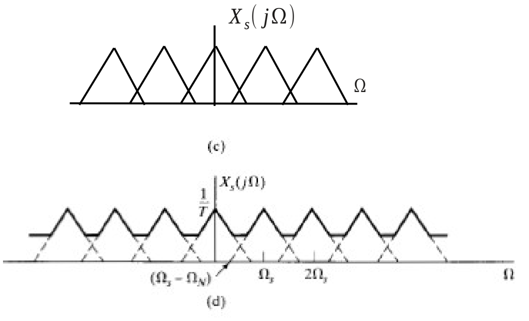
$\omega=\Omega T$

Bibliografía: Opp 4.2

Problemas Opp: 4.2*, 4.3*, 4,4*.

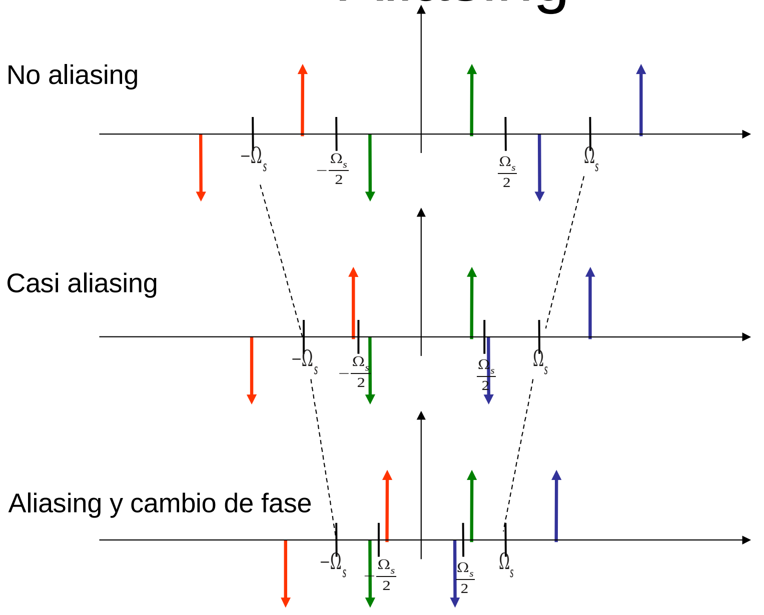
Aliasing

Se produce aliasing cuando $\Omega_N > \Omega_s - \Omega_N$ (1.7)



Bibliografía: Opp 4.2

Aliasing



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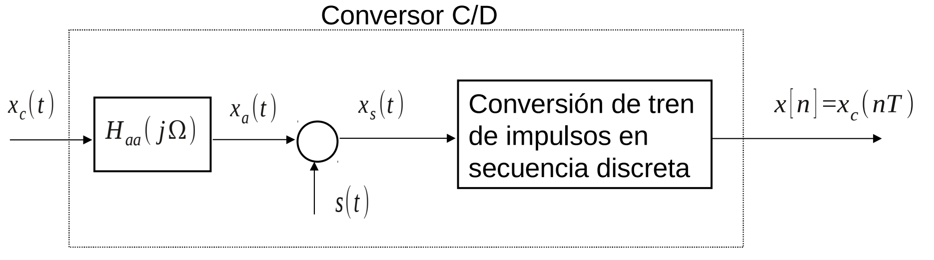
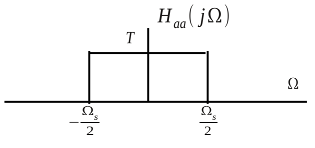
Bibliografía: Opp 4.2

Filtro Anti-aliasing





Bibliografía: Opp 4.2

Problemas Opp: 4.5*, 4.8*

Reconstrucción de la señal

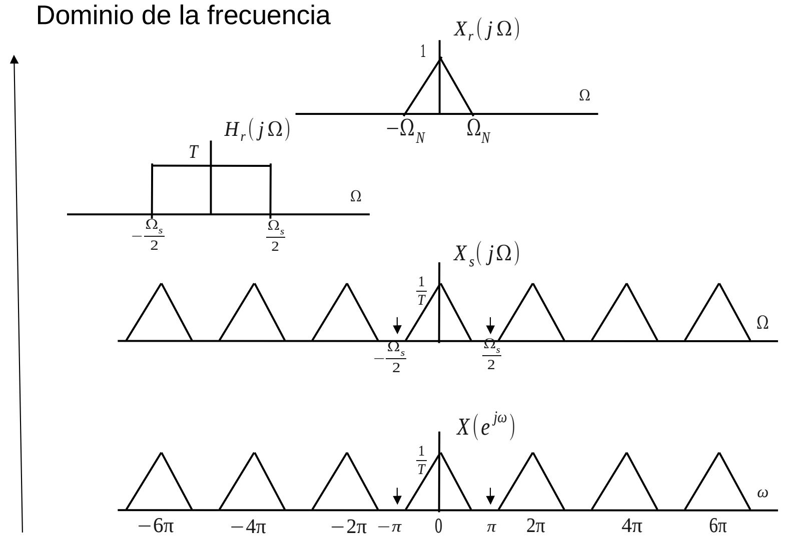
Dominio del tiempo

$x_r(t) = x_s(t) * h_r(t)$

$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - nT)$

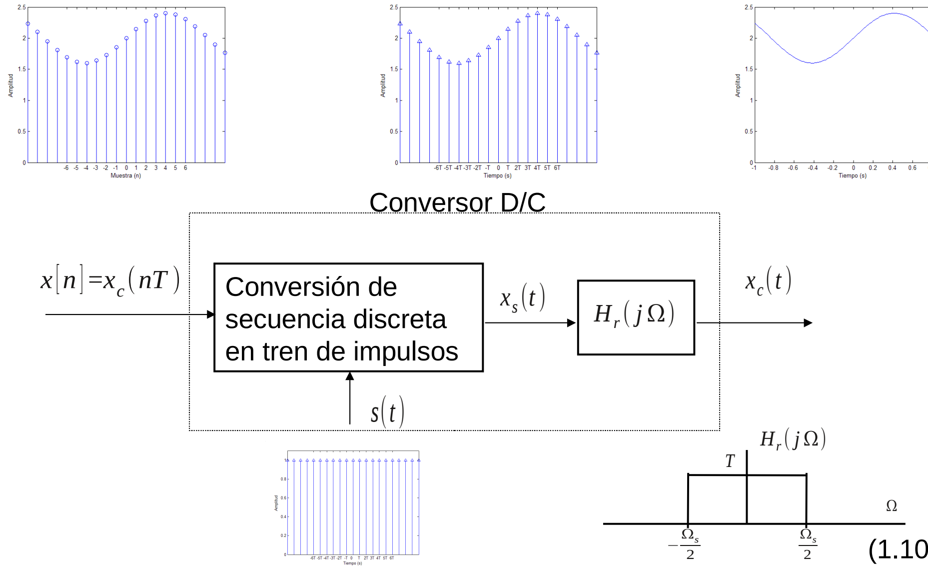
$x[n] = x_c(nT)$

Dominio de la frecuencia



Bibliografía: Opp 4.3

Reconstrucción de la señal



(1.10)

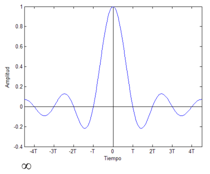
Bibliografía: Opp 4.3

Reconstrucción de la señal

Dominio del tiempo

$$x_r(t) = x_s(t) * h_r(t) = \sum_{n=-\infty}^{\infty} x[n] h_r(t - nT) \quad (1.11)$$

$$h_r(t) = \operatorname{sinc}\left(\frac{t}{T}\right)$$



$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT) \quad (1.13)$$

$$x[n] = x_c(nT) \quad (1.15)$$

Dominio de la frecuencia

$$X_r(j\Omega) = H_r(j\Omega) X_s(j\Omega) = H_r(j\Omega) X\left(e^{j\Omega T}\right) \quad (1.12)$$

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega nT} = X\left(e^{j\Omega T}\right) \quad (1.14)$$

[1.3]

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad (1.16)$$

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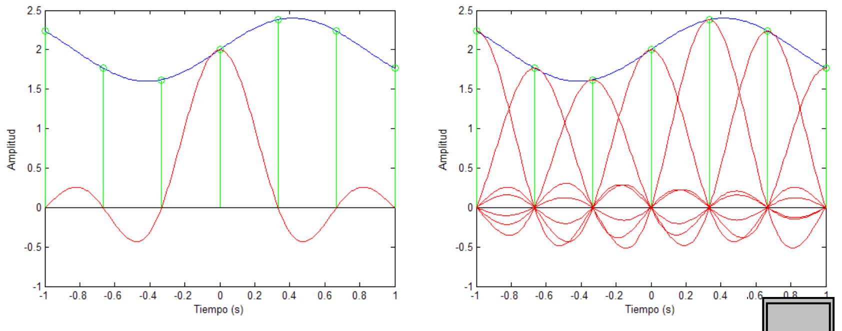
Bibliografía: Opp 4.3

Reconstrucción de la señal

$$x_r(t) = x_s(t) * h_r(t) = \sum_{n=-\infty}^{\infty} x[n] h_r(t - nT) = \sum_{n=-\infty}^{\infty} x[n] \frac{sen[\pi(t-nT)/T]}{\pi(t-nT)/T}$$

$$h_r(t) = \sin c\left(\frac{t}{T}\right)$$



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Bibliografía: Opp 4.3

Problemas Opp: 4.19

Problemas Pro: 9.12

Teorema del muestreo (1.17)

Sea una señal $x_c(t)$ limitada en ancho de banda cuya frecuencia máxima es $f_{\max}$. Entonces, esta señal se puede recuperar exactamente a partir de sus muestras tomadas a una frecuencia $\frac{1}{T}=f_s\geq 2\mathrm{f}_{\max}$ mediante la función de interpolación $h_r(t)=\sin c\left(\frac{t}{T}\right)$ .La fórmula correspondiente de interpolación es $x_r(t)=x_s(t)*h_r(t)=\sum_{n=-\infty}^{\infty} x[n]h_r(t-nT)$

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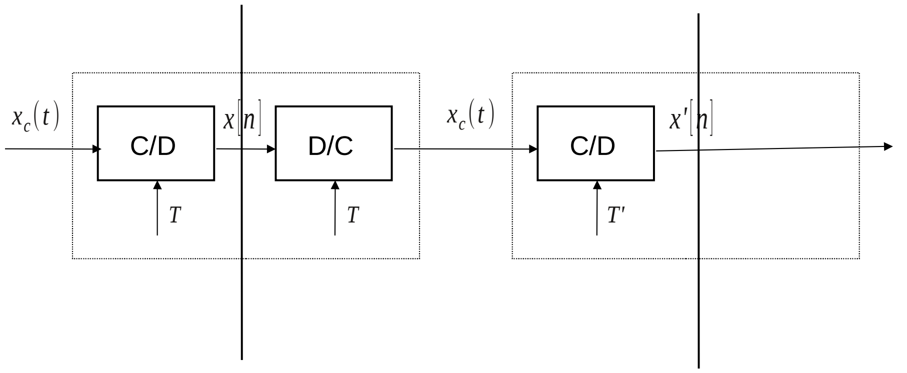
Bibliografía: Proakis, Pag 28

Cambio de la Frecuencia de Muestreo

Cambio de la frecuencia de muestreo



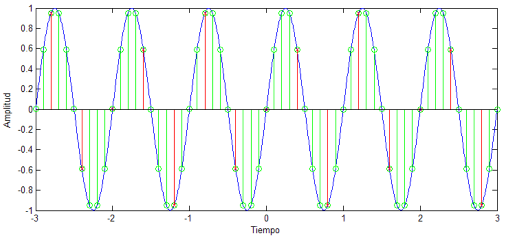
$x[n] = x(nT)$

$x'[n] = x(nT')$

Bibliografía: Opp 4.6

Reducción de la frecuencia de muestreo

$x[n]$ → ↓M → $x_d[n]=x[nM]=x_c(nMT) \Rightarrow T'=MT$ (1.21)



[1.4] $X(e^{j\omega})=\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\frac{\omega}{T}-k\frac{2\pi}{T}))$

$r=i+kM$

$-\infty<k<\infty$

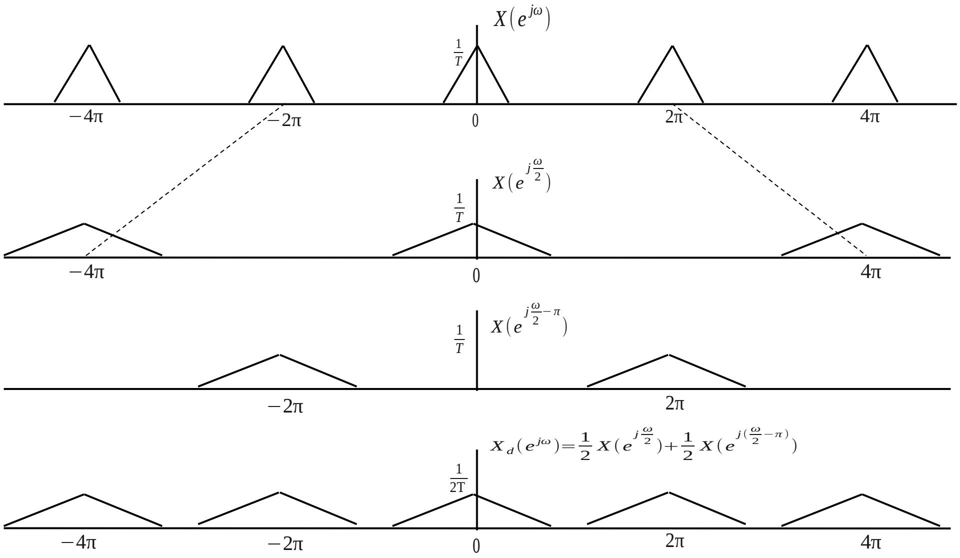
$0\leq i\leq M-1$

$$\boxed{X_d(e^{j\omega})}=\frac{1}{T'}\sum_{r=-\infty}^{\infty} X_c(j(\frac{\omega}{T'}-r\frac{2\pi}{T'}))=\frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c(j(\frac{\omega}{MT}-r\frac{2\pi}{MT}))=$$

$$=\frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\frac{\omega}{MT}-k\frac{2\pi}{T}-i\frac{2\pi}{MT}))=\boxed{\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)} \quad (1.22)$$

Bibliografía: Opp 4.6

Reducción de la frecuencia de muestreo



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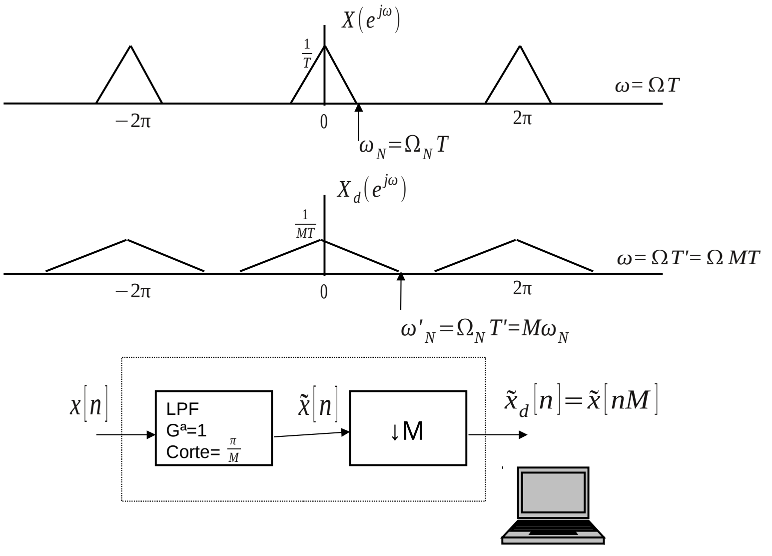
Bibliografía: Opp 4.6

Reducción de la frecuencia de muestreo



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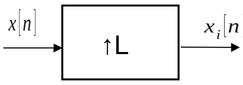
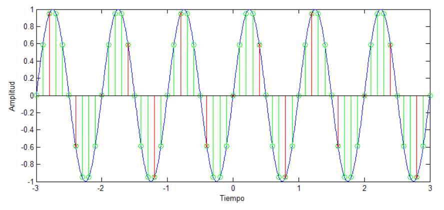
Bibliografía: Opp 4.6

Problemas Opp: 4.9*, 4.14, 4.26, 4.36, 4.46

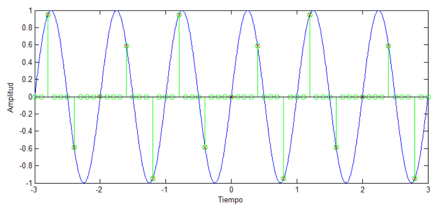
Incremento de la frecuencia de muestreo



$x_i[n] = x_c(nT/L) \Rightarrow T' = T/L$ (1.23)

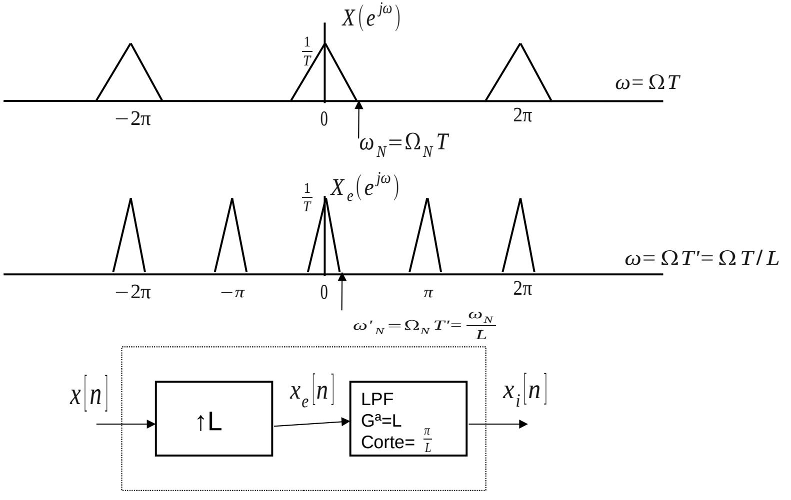


(1.24) $x_e[n] = \begin{cases} x[n/L] & n = 0, \pm L, \pm 2L, \ldots \\ 0 & resto \end{cases} = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] \longleftrightarrow X_e(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k] e^{j\omega kL} = X(e^{j\omega L})$ (1.25)



Bibliografía: Opp 4.6

Incremento de la frecuencia de muestreo



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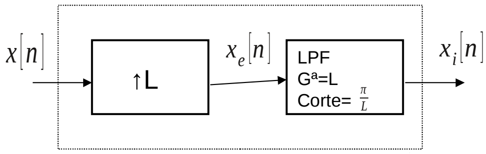
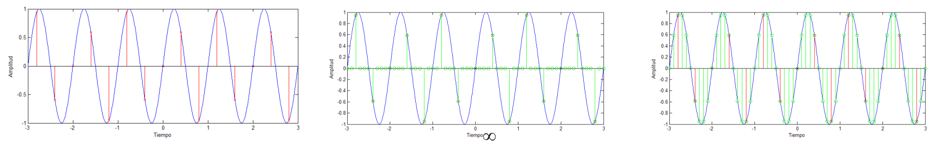
Bibliografía: Opp 4.6

Incremento de la frecuencia de muestreo

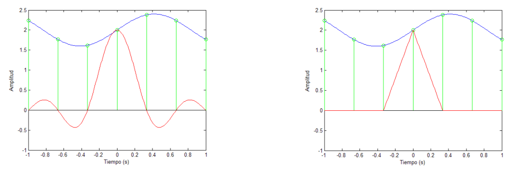




$$x_i[n]=x_e[n]*h_i[n]=\sum_{k=-\infty}^{\infty} x_e[kL]\,h_i[n-kL] \quad (1.26)$$

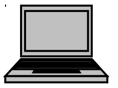
$$h_i[n]=\mathrm{sinc}\left(\frac{n}{L}\right)$$

(1.27)



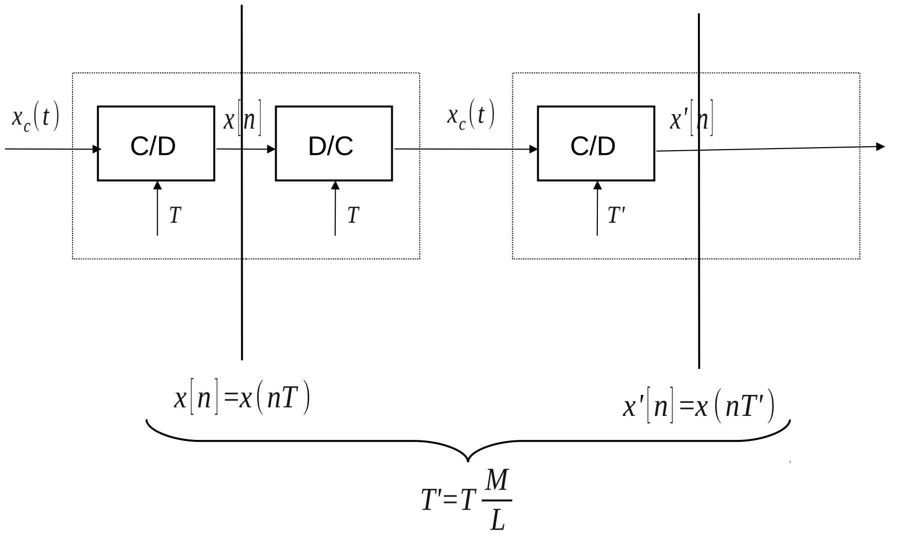
$$h_i[n]=\begin{cases}1-\dfrac{|n|}{L} & |n|<L\\ 0 & resto\end{cases}$$

(1.28)



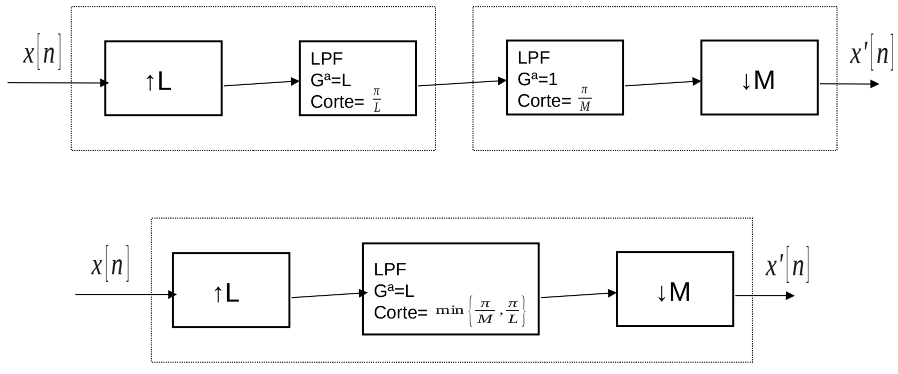
Bibliografía: Opp 4.6

Cambio de la frecuencia de muestreo por un factor racional



Bibliografía: Opp 4.6

Cambio de la frecuencia de muestreo por un factor racional

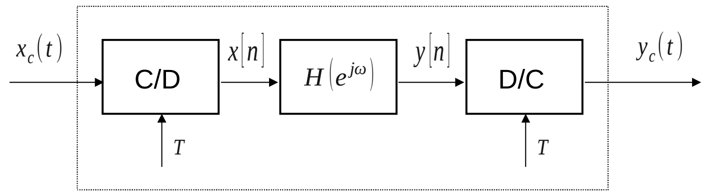


Bibliografía: Opp 4.6

Problemas Opp: 4.16*, 4.17*, 4.18, 4.38*, 4.40*, 4.41, 4.42, 4.44, 4.51, 4.52, 4.53, 4.54, 4.55, 4.58, 4.59, 4.60

Procesado discreto de señales continuas

Procesado discreto de señales continuas



[1.4] $X(e^{j\omega})=\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T}-k\frac{2\pi}{T}\right)\right)$ → $Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})$ → $Y_c(j\Omega)=H_r(j\Omega)Y(e^{j\Omega T})$ [1.12]

$$Y_c(j\Omega)=H_{eff}(j\Omega)X_c(j\Omega)$$

$$H_{eff}(j\Omega)=\begin{cases} H(e^{j\Omega T}) & |\Omega T|<\pi \\ 0 & resto \end{cases} \quad (1.18)$$

$$H(e^{j\omega})=H_{eff}\left(j\frac{\omega}{T}\right) \quad \forall \omega : |\omega|<\pi \quad (1.19)$$

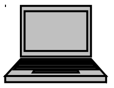
Invarianza de la respuesta al impulso

$$h[n]=Th_{eff}(nT) \quad (1.20)$$

[1.4]

Bibliografía: Opp 4.4

Procesado discreto de señales continuas



Ejemplo:

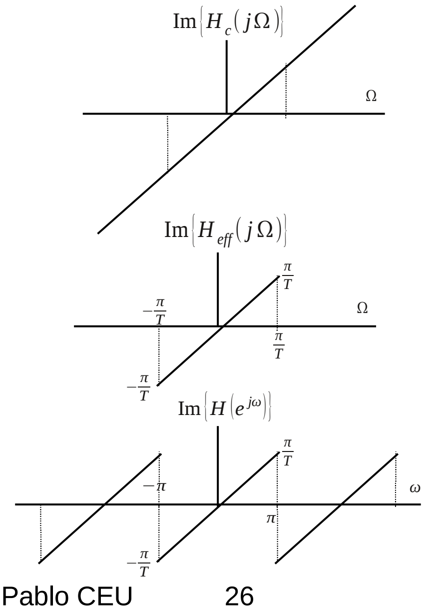
$$y_c(t) = \frac{dx_c(t)}{dt}$$

$$H_c(j\Omega) = j\Omega$$

$$H_{eff}(j\Omega) = \begin{cases} j\Omega & |\Omega T| < \pi \\ 0 & resto \end{cases}$$

$$H(e^{j\omega}) = j\frac{\omega}{T} \quad \forall\, \omega : |\omega| < \pi$$

[0.125] $$h[n] = \begin{cases} 0 & n = 0 \\ \dfrac{\cos \pi n}{nT} & n \neq 0 \end{cases}$$

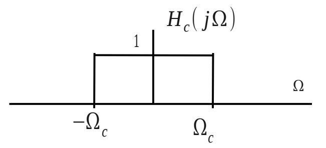


Bibliografía: Opp 4.4

Procesado discreto de señales continuas

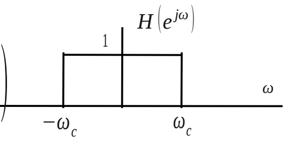
Ejemplo:

$$H_c(j\Omega) = H_{eff}(j\Omega) = \begin{cases} 1 & |\Omega| < \Omega_c : \Omega_c T < \pi \\ 0 & resto \end{cases}$$



$$h(t) = \frac{\Omega_c}{\pi} \operatorname{sinc}\left(\frac{\Omega_c}{\pi} t\right)$$

$$h[n] = Th(nT) = T\frac{\Omega_c}{\pi} \operatorname{sinc}\left(\frac{\Omega_c}{\pi} nT\right) = \frac{\omega_c}{\pi} \operatorname{sinc}\left(\frac{\omega_c}{\pi} n\right)$$



$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c : \omega_c < \pi \\ 0 & resto \end{cases} \qquad \begin{matrix} \omega_c = \Omega_c T \\ -\pi \le \omega < \pi \end{matrix}$$

[1.11]

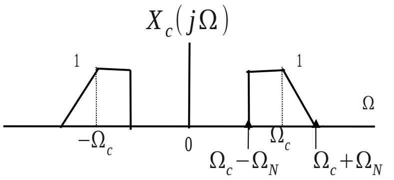
Bibliografía: Opp 4.4

Problemas Opp: 4.12, 4.13, 4.20*, 4.22, 4.23, 4.24*, 4.25, 4.28*, 4.30, 4.31, 4.32, 4.33, 4.34, 4.35, 4.37, 4.45, 4.49

Problemas Pro: 9.4, 9.5

Muestreo de señales paso banda

Muestreo de señales paso banda



$\Omega_s \geq 2(\Omega_c+\Omega_N)$!!

Solución: Muestrear el equivalente paso bajo

[0.153] $x(t)=x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t$

El ancho de banda de la señal equivalente paso bajo es Ω_N .
La idea es muestrear la componente en fase y en cuadratura por separado

Supongamos que $\Omega_c+\Omega_N = k(2\Omega_N)$ (1.52)

$\Omega_s = 2(2\Omega_N)$ (1.53)

Nyquist

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Bibliografía: Proakis 9.1

Hacer aquí Opp 4.21

Muestreo de señales paso banda

[0.153] $x(t) = x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t$

(1.54) $x[n] = x(nT) = x_c(nT)\cos\Omega_c nT - x_s(nT)\sin\Omega_c nT =$ $\longleftarrow$ $\begin{cases} \Omega_c + \Omega_N = k(2\Omega_N) \\ \Omega_s = 2(2\Omega_N) = \frac{2\pi}{T} \end{cases}$

$= x_c(nT)\cos\left(\pi n \frac{2k-1}{2}\right) - x_s(nT)\sin\left(\pi n \frac{2k-1}{2}\right) =$

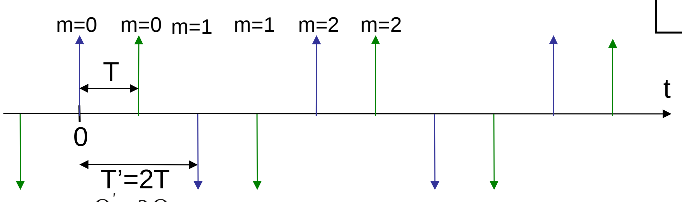
[1.52,1.53]

$$= \begin{cases} x_c(mT')(-1)^m & n = 2m \\ (-1)^{k+1} x_s(mT' - T)(-1)^m & n = 2m-1 \end{cases}$$

$\Omega_c + \Omega_N = k(2\Omega_N) \longrightarrow \Omega_N = \frac{\Omega_c}{2k-1}$

$\Omega_s = 2(2\Omega_N) = \frac{2\pi}{T} \longrightarrow \Omega_N = \frac{\pi}{2T}$

$\frac{\Omega_c}{2k-1} = \frac{\pi}{2T} \Rightarrow \Omega_c T = \pi\frac{2k-1}{2}$



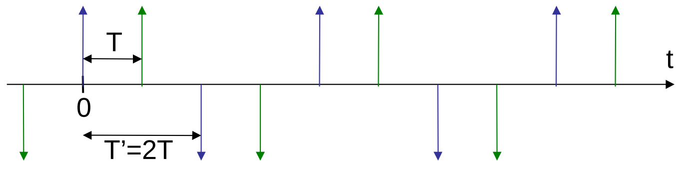
$\Omega_s' = 2\Omega_N$

(1.55)

Bibliografía: Proakis 9.1

Muestreo de señales paso banda

Reconstrucción de cada una de las componentes



$x[0]=x_c[0](-1)^0$
$x[1]=x_s[0](-1)^{k+1}$
$x[2]=x_c[1](-1)^1$
$x[3]=x_s[1](-1)^{1+k+1}$
$x[4]=x_c[2](-1)^2$
$x[5]=x_s[2](-1)^{2+k+1}$

(1.56) $x_c[m]=x[2m](-1)^m$ $\quad$ $x_s[m]=x[2m+1](-1)^{m+k+1}$ (1.57)

(1.58) $x_c(t)=\sum_{m=-\infty}^{\infty} x_c[m]h_r(t-mT')$ $\quad$ $x_s(t)=\sum_{m=-\infty}^{\infty} x_s[m]h_r(t-(mT'+T))$ (1.59)

Bibliografía: Proakis 9.1

Muestreo de señales paso banda

[0.153,1.58,1.59]

$$\boxed{x(t)} = x_c(t)\cos\Omega_c t - x_s(t)\sin\Omega_c t =$$

$$= \sum_{m=-\infty}^{\infty} \left(x_c[m]h_r(t-mT')\cos\Omega_c t - x_s[m]h_r(t-(mT'+T))\sin\Omega_c t \right) =$$

$$= \sum_{m=-\infty}^{\infty} \left(x[2\mathrm{m}](-1)^m h_r(t-mT')\cos\Omega_c t - x[2\mathrm{m}+1](-1)^{m+k+1} h_r(t-(mT'+T))\sin\Omega_c t \right) =$$

$$\cos(a+b) = \cos a\cos b + \sin a \sin b$$

$$\Omega_c T = \pi\frac{2\mathrm{k}-1}{2}$$

$$= \sum_{m=-\infty}^{\infty} \left(x[2\mathrm{m}]h_r(t-2mT)\cos\Omega_c(t-2mT) - x[2\mathrm{m}+1]h_r(t-(2\mathrm{m}+1)T)\cos\Omega_c(t-(2\mathrm{m}+1)T) \right) =$$

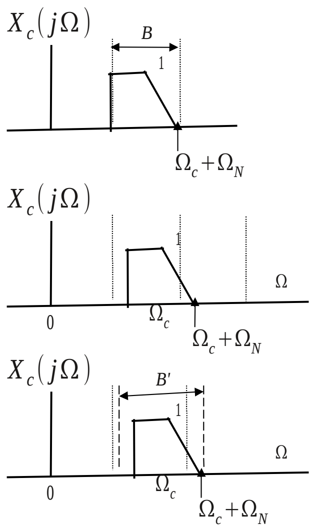
$$\boxed{= \sum_{n=-\infty}^{\infty} x[n]h_r(t-nT)\cos\Omega_c(t-nT)}$$

Suponiendo que $\Omega_c + \Omega_N = k(2\Omega_N)$ [1.52]

(1.60)

Bibliografía: Proakis 9.1

Muestreo de señales paso banda



Supongamos que $\Omega_c+\Omega_N = kB$

Supongamos que $\Omega_c+\Omega_N \neq kB$ (1.61)

Entonces $k=\left\lfloor \frac{\Omega_c+\Omega_N}{B}\right\rfloor$ (1.62)

$$B'=2\Omega'_N=\frac{\Omega_c+\Omega_N}{k} \quad (1.63)$$

$$\Omega'_c+\Omega'_N=kB' \quad (1.64)$$

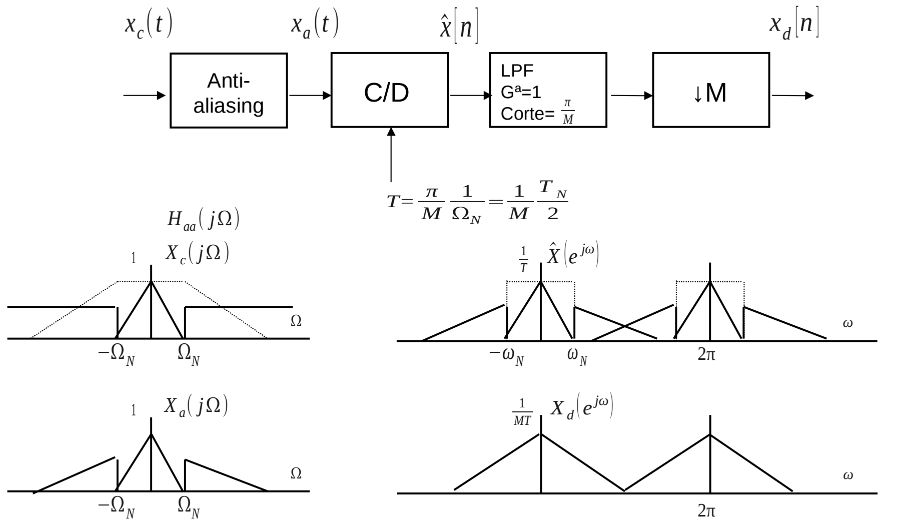
$$2(2\Omega_N)\le\Omega'_s<4(2\Omega_N) \quad (1.65)$$

Bibliografía: Proakis 9.1

Problemas Pro: 9.1, 9.2, 9.3, 9.13

Aplicaciones del Oversampling

Aplicaciones del oversampling: Filtros antialiasing



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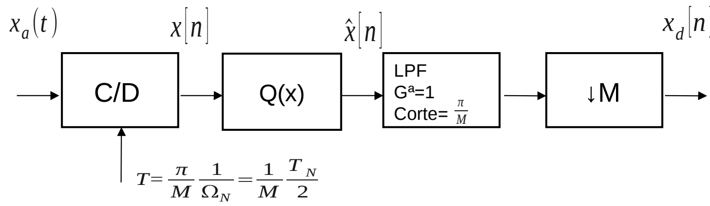
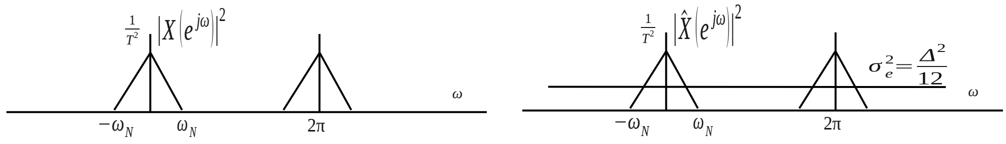
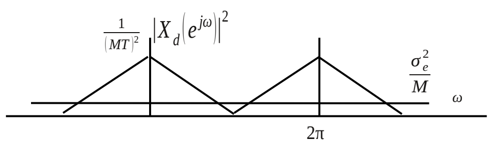
Bibliografía: Opp 4.9

Aplicaciones del oversampling: Reducción del ruido de muestreo

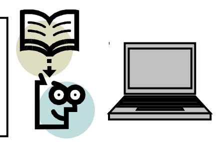






$$E\{x_d^2[n]\} = E\{x^2[n]\}$$

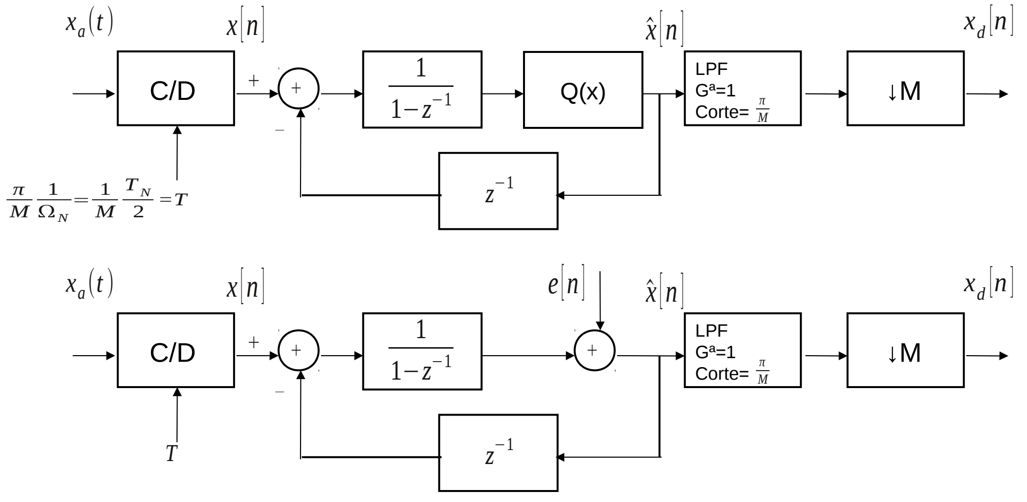
$$E\{e_d^2[n]\} = \frac{1}{M} E\{e^2[n]\}$$



(1.66, 1.67)

Bibliografía: Opp 4.9

Aplicaciones del oversampling: Noise shaping



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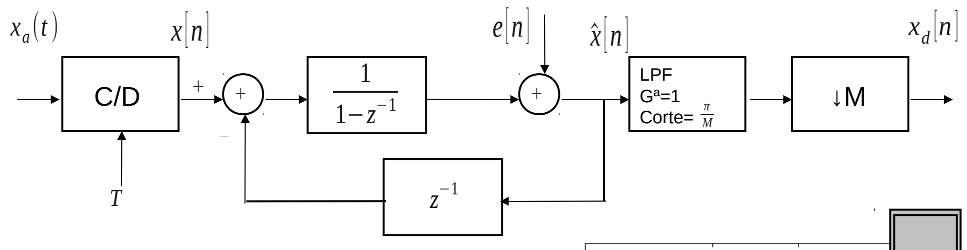
Bibliografía: Opp 4.9

Aplicaciones del oversampling: Noise shaping



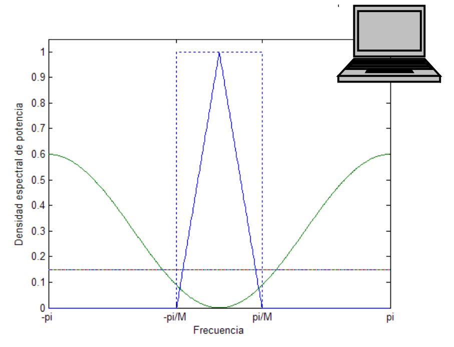
$$\hat{X}(z)=(X(z)-z^{-1}\hat{X}(z))\frac{1}{1-z^{-1}}+E(z)$$

(1.68) $\hat{X}(z)=X(z)+(1-z^{-1})E(z)$

(1.69) $\hat{x}[n]=x[n]+e[n]-e[n-1]=x[n]+e'[n]$

$\sigma_{e'}^2=2\sigma_e^2$ (1.70)

(1.71) $S_{e'}(e^{j\omega})=|1-e^{-j\omega}|^2\sigma_e^2=\left(2\sin\frac{\omega}{2}\right)^2\sigma_e^2$



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Bibliografía: Opp 4.9

Problemas Opp: 4.61, 4.62

Problemas Pro: 9.8